DBD within Continuum-QRPA: first preliminary results

Vadim Rodin

Eberhard Karls
Universität
Tübingen

INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS

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Outline

- Introduction
- Motivation for continuum QRPA
- Basics of continuum QRPA
- DBD within continuum QRPA
- Calculations results
- Conclusions

Inverse Half-Lives $[T_{1/2}^{-1}(0^+ \to 0^+)]$

$$G^{2\nu}(E_0, Z)\, |M_{GT}^{2\nu}|^2$$

$$\langle m_\nu \rangle^2\, G^{0\nu}(E_0, Z)\, \left|M^{0\nu}\right|^2$$

Eff. neutrino mass $\langle m_\nu \rangle = \sum_j m_j U_{ej}^2$

Introduction

Nuclear structure models to calculate $\beta\beta$-amplitudes

Mean field $\longrightarrow$ s.p. states $\longrightarrow$ diagonalization

Quasiparticle Random Phase Approximation
versus Nuclear Shell Model

	QRPA	NSM
s.p. states	all ($N\hbar\omega$)	limited ($0\hbar\omega$)
complex states	limited (g.s. correlations)	all

$0\hbar\omega$ model space of NSM is not enough for $0\nu\beta\beta$

Introduction

Our recent works[a] – the most advanced QRPA calculations up-to-date.

Is it "the final result"?

No, the calculations — within the "standard QRPA"

How accurate is the "standard QRPA" itself?
Are not there some important contributions missing?

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[a] V. R., A. Faessler, F. Simkovic, P. Vogel, PRC **68** (2003); nucl-th/0503063

Motivation for continuum-QRPA

$\Rightarrow$ to perform ultimate QRPA calculations

- To include entire single-particle basis — no more question about the dependence of the QRPA results on the s.p.-basis size
- To use realistic wave functions of the continuum states – no more need for oscillator wave functions
- To get a new insight into the results via alternative formulation of the QRPA

Motivation for cQRPA

Usual statement

"In the QRPA one can include essentially an unlimited set of single-particle states

but only a limited subset of configurations (iterations of the particle-hole configurations)."

Correct, but in quite ideal situation

large number of the active shells in the calculations $N \gg 1$
$\Rightarrow$ only by adding low-lying shells, "core" of bound s.p. states.

to include high-lying shells
$\Rightarrow$too severe approximation of continuum states by discrete ones
Realistically only one shell lying higher than the Fermi-level one can be considered

Motivation for cQRPA

Role of the continuum:

- Pairing in the continuum can increase $0\nu\beta\beta$ sum rules ($M^{0\nu}\uparrow$)
- Additional g.s. correlations can appear due to collective states in the continuum ($M^{0\nu}\downarrow$)

At the moment — no continuum-QRPA including consistently pairing in the continuum.

Separately - both are known: cQRPA with pairing on discrete basis and pairing equations in continuum.

Basics of cQRPA

$$|JM\rangle = Q^{\dagger}_{JM}|0^{+}_{RPA}\rangle \qquad Q^{\dagger}_{JM} = \sum_{12}\left[X_{12}A^{\dagger}_{12} - Y_{12}\tilde{A}_{12}\right]$$

It is not possible to handle the infinite number of the QRPA amplitudes X_s, Y_s if one wants to include the single-particle continuum.

Instead, the QRPA is reformulated in terms of four transition densities $\varrho_i^{J^\pi s}$ $(i = 1, \ldots 4)$ of the excited states in the coordinate space

The continuum-RPA has been used for a long time to successfully describe structure and decay properties of various giant resonances and their high-lying overtones embedded in the single-particle continuum

Basics of cQRPA

Elements $\varrho_i^{J^\pi s}(r)$ in terms of the QRPA amplitudes $X_{\pi\nu}^{J^\pi s}$ and $Y_{\pi\nu}^{J^\pi s}$

$$\varrho_i(r) = \sum_{\pi\nu} t^{(J)}_{(\pi)(\nu)} \chi_\pi(r) \chi_\nu(r) R_i^{\pi\nu},$$

$$\begin{pmatrix} R^{\pi\nu}_{p-h} \\ R^{\pi\nu}_{h-p} \\ R^{\pi\nu}_{p-p} \\ R^{\pi\nu}_{h-h} \end{pmatrix} = \begin{pmatrix} u_\pi v_\nu X_{\pi\nu} + v_\pi u_\nu Y_{\pi\nu} \\ u_\pi v_\nu Y_{\pi\nu} + v_\pi u_\nu X_{\pi\nu} \\ u_\pi u_\nu X_{\pi\nu} - v_\pi v_\nu Y_{\pi\nu} \\ u_\pi u_\nu Y_{\pi\nu} - v_\pi v_\nu X_{\pi\nu} \end{pmatrix}$$

$t^{(J)}_{(\pi)(\nu)} = \frac{1}{\sqrt{2J+1}} \langle \pi \| T_{JLS} \| \nu \rangle$ – reduced m. e. of the spin-angular tensor $T_{JLS\mu}$

u, v - coefficients of Bogolyubov transformation

M. e. of a probing s.p. operator $\hat{V}^{(-)}_{J\mu} = \sum_a V_J(r_a)\, T_{JLS\mu}\, \tau^-_a$ between $|0\rangle$ and $|s, J\mu\rangle$:

$$V^{(-)}_{s0} = \int \varrho_1^{J^\pi s}(r) V_J(r)\, dr$$

The pn-QRPA system of equations for ϱ_i

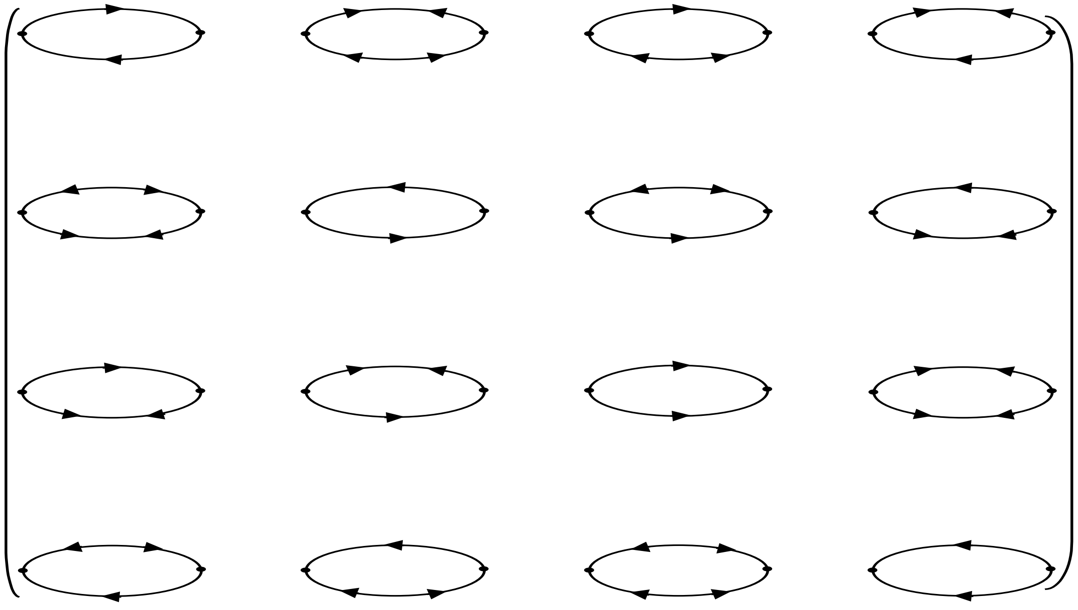
$$\varrho = \{AF\varrho\}$$

$$\varrho_i^{J^\pi s}(r) = \sum_k \int A_{ik}^{J^\pi}(rr', \omega = \omega_s)\, F_k^{J^\pi}(r'r'')\, \varrho_k^{J^\pi s}(r'')\, dr'dr'',$$

$F_k^{J^\pi}(r_1 r_2)$ — residual interaction in k-channel after separation of spin-angular variables

Free two-quasiparticle propagator A_{ik}

in terms of normal and anomalous single-particle Green's functions for Fermi-systems with nucleon pairing



Basics of cQRPA

$$A^J_{ik}(r_1 r_2, \omega) = \sum_{\pi\nu} \left(t^{(J)}_{(\pi)(\nu)}\right)^2 \chi_\pi(r_1)\chi_\nu(r_1)\,\chi_\pi(r_2)\chi_\nu(r_2)\,A^{\pi\nu}_{ik}(\omega)$$

$$A^{\pi\nu}_{11} = \frac{u_\pi^2 v_\nu^2}{\omega - E_\pi - E_\nu} - \frac{u_\nu^2 v_\pi^2}{\omega + E_\pi + E_\nu}, A^{\pi\nu}_{12} = \frac{u_\pi v_\pi v_\nu u_\nu}{\omega - E_\pi - E_\nu} - \frac{u_\pi v_\pi v_\nu u_\nu}{\omega + E_\pi + E_\nu}$$

The way to explicitly take the s.p. continuum into consideration:

1. To put for highly-excited s.p. states: $v = 0,\ u = 1,$
   $E = |\varepsilon - \lambda|\ (\gg \Delta)$ (accuracy $\left(\frac{\Delta}{|\varepsilon-\lambda|}\right)^2$)
2. To use the s.p. Green function: $g(r_1 r_2, \varepsilon) = \sum_\pi \frac{\chi_\pi(r_1)\chi_\pi(r_2)}{\varepsilon-\varepsilon_\pi}$
   to perform summation over the s.p. states in continuum

Basics of cQRPA

Total two-quasiparticle propagator $\hat{A}$

including QRPA iterations of p-h and p-p interactions

$\Rightarrow$ a Bethe-Salpeter-type integral equation:

$$\hat{A} = A + \{AF\hat{A}\}$$

Spectral decomposition:

$$\hat{A}_{ik}^{J^\pi}(r_1 r_2, \omega) = \sum_s \frac{\varrho_i^{J^\pi s}(r_1)\varrho_k^{J^\pi s}(r_2)}{\omega - \omega_s + i\delta} - \sum_s \frac{\varrho_i^{J^\pi s}(r_1)\varrho_k^{J^\pi s}(r_2)}{\omega + \omega_s - i\delta}$$

DBD within cQRPA

The strength function for a single-particle probing operator $\hat{V}^{(\mp)}_{J\mu}$ defined by

$$S^{(\mp)}(\omega) = \sum_s \left| \langle s | \hat{V}^{(\mp)}_{J\mu} | 0 \rangle \right|^2 \delta(\omega - \omega_s)$$

can be calculated in term of Im $\hat{A}$:

$$S^{(-)}(\omega) = -\tfrac{1}{\pi} \, \mathrm{Im} \int V_J(r_1) \hat{A}^J_{11}(r_1 r_2; \omega) V_J(r_2) \, dr_1 dr_2$$

$$S^{(+)}(\omega) = -\tfrac{1}{\pi} \, \mathrm{Im} \int V_J(r_1) \hat{A}^J_{22}(r_1 r_2; \omega) V_J(r_2) \, dr_1 dr_2$$

DBD within cQRPA

Non-diagonal strength function

$$S^{(--)}(\omega) = \sum_s \langle 0'|\hat{V}^{(-)}_{J\bar{\mu}}|s\rangle\langle s|\hat{V}^{(-)}_{J\mu}|0\rangle\delta(\omega - \omega_s)$$

Identifying BCS vacuum $|0'\rangle$ with that of $|0\rangle$

$$S^{(--)}(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\int V_J(r_1)\hat{A}^J_{12}(r_1 r_2;\omega)V_J(r_2)\,dr_1 dr_2$$

$$M^{2\nu}_{GT} = -\frac{3}{2}\int \hat{A}^1_{12}(r_1 r_2;\omega=0)\,dr_1 dr_2 \quad + \quad \delta M^{2\nu}_{GT}$$

$$\delta M^{2\nu}_{GT} = -\frac{1}{\pi}\int d\omega\left(\frac{1}{\omega+\delta E} - \frac{1}{\omega}\right)\int \mathrm{Im}\,\hat{A}^1_{12}(r_1 r_2;\omega)\,dr_1 dr_2$$

DBD within cQRPA

The same procedure can be applied to calculate within the cQRPA the matrix element of a 2-body operator

$$\hat{V}_2^{(--)} = \sum_{ab} \sum_{JLS} V_J(r_a, r_b) T_{JLS\mu}(n_a) T^*_{JLS\mu}(n_b) \tau_a^{(-)} \tau_b^{(-)}$$

between the ground states $|0\rangle$ and $|0'\rangle$

$$\langle 0'|\hat{V}_2^{(--)}|0\rangle = \sum_J -\frac{1}{\pi} \int d\omega \int V_J(r_1, r_2)\, \mathrm{Im}\, \hat{A}^J_{12}(r_1 r_2; \omega)\, dr_1 dr_2$$

Some preliminary results

Model

Landau-Migdal zero-range pairing, p-h and p-p forces
(all strengths – in units of 300 MeV$\cdot$ fm^3)

Input parameters

pairing strengths g_n^{pair}, g_p^{pair}

p-h strengths: isovector f_{ph}^0 and spin-isovector f_{ph}^1

$$(f_{ph}^0 + f_{ph}^1 \sigma_\mathbf{a} \sigma_\mathbf{b}) \tau_\mathbf{a} \tau_\mathbf{b} \delta(\mathbf{r_a} - \mathbf{r_b})$$

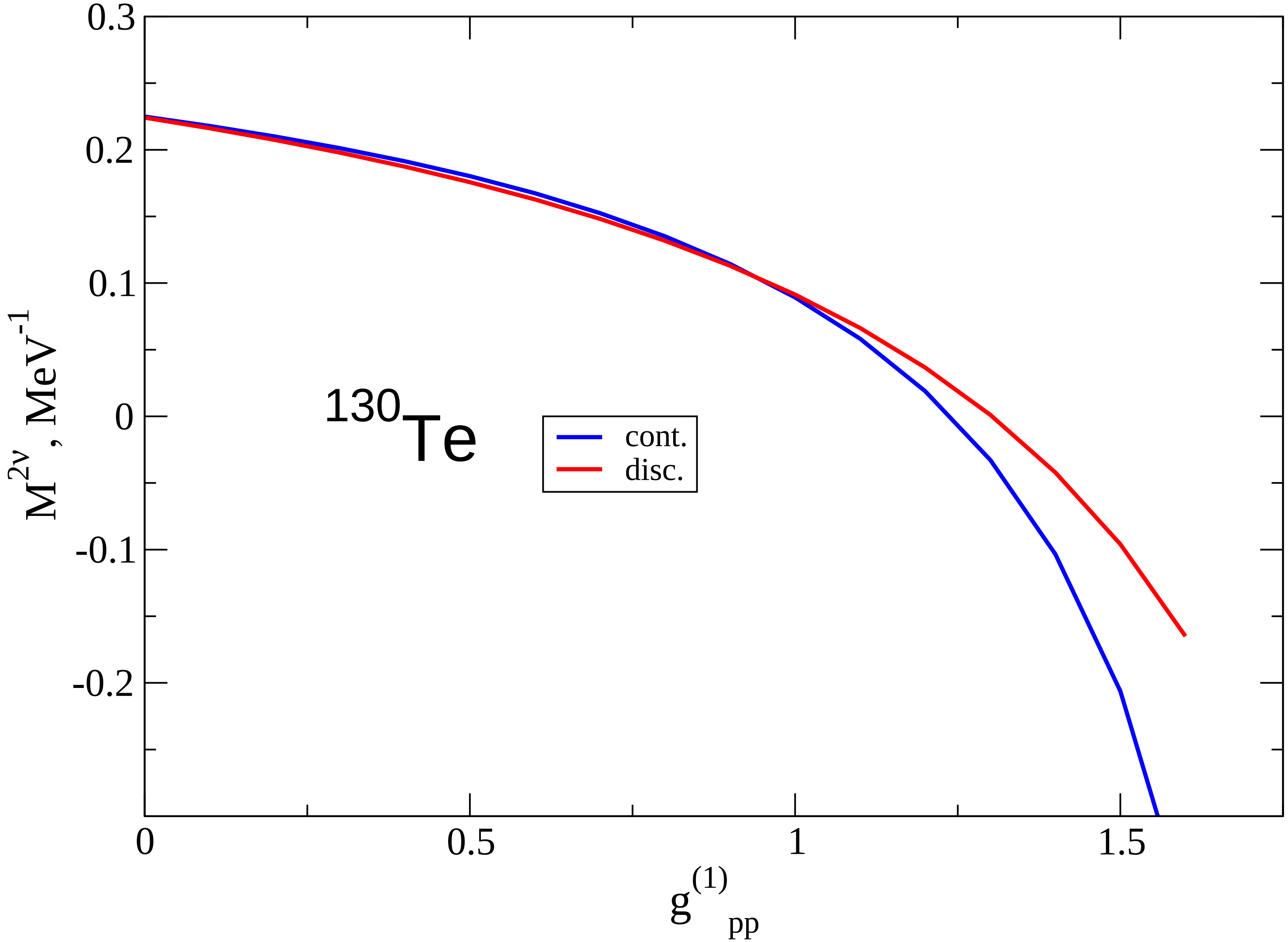
p-p strengths g_{pp}^0, g_{pp}^1

Some preliminary results

Fixing input parameters

- g^{pair} – to reproduce exp. pairing energies
- $f^0_{ph} = 1.0$ – isospin-selfconsistency of p-h interaction and mean field
- f^1_{ph} – to reproduce the exp. energy of the GTR
- $g^0_{pp} = (g^{pair}_n + g^{pair}_p)/2$ – isospin-selfconsistency of p-p interaction
- g^1_{pp} – to reproduce exp. $M^{2\nu}$

  Absolute value of g^1_{pp} - non-informative, depends on basis size. Relative strength $2g^1_{pp}/(g^{pair}_n + g^{pair}_p)$ should be used instead (a specific sum rule is governed by this ratio)



0.3
0.2
0.1
0
-0.1
-0.2
0
0.5
1
1.5
$M^{2\nu}$, MeV^{-1}
$g^{(1)}_{pp}$
^{130}Te
cont.
disc.

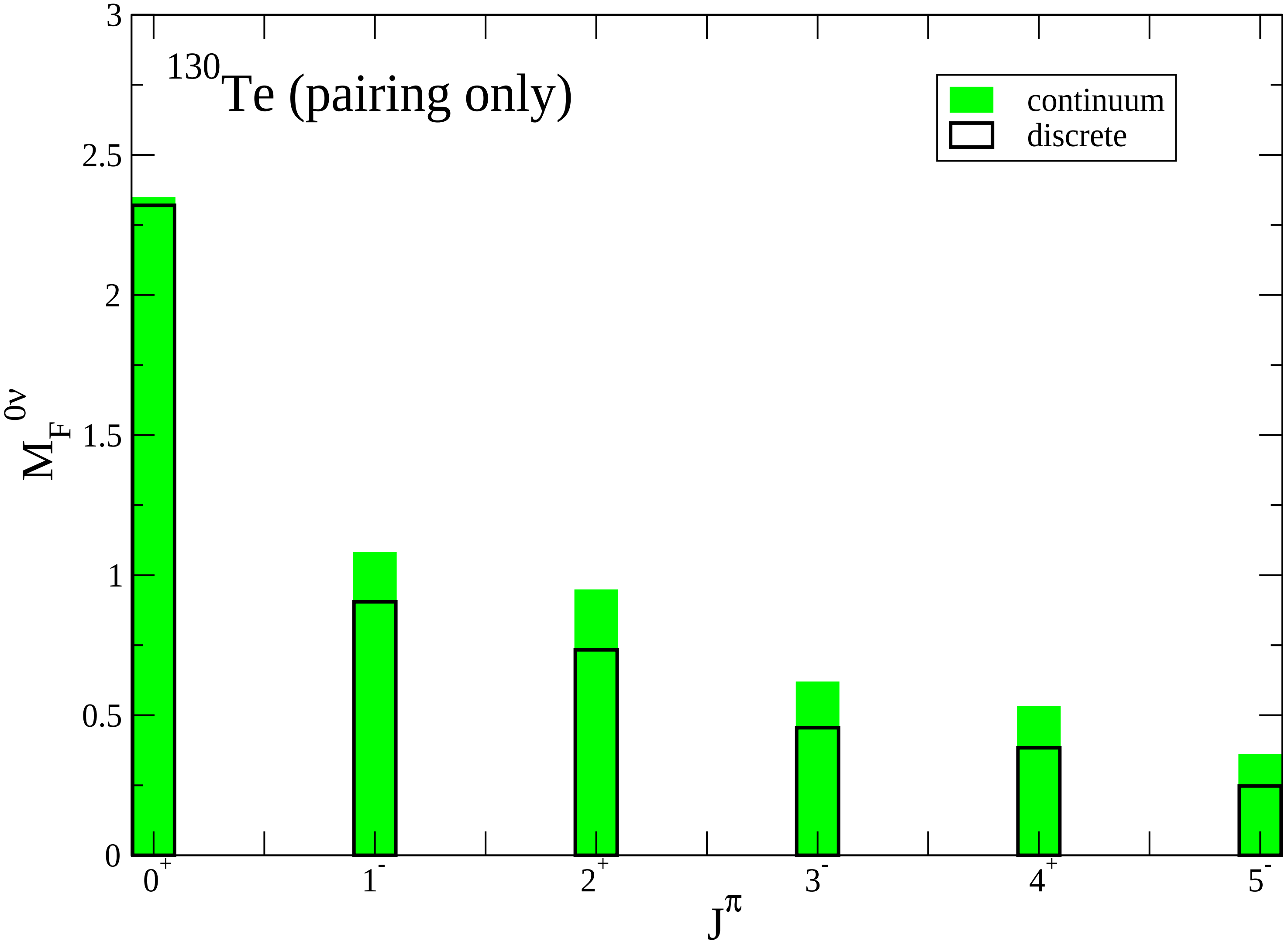
Some preliminary results

^{130}Te

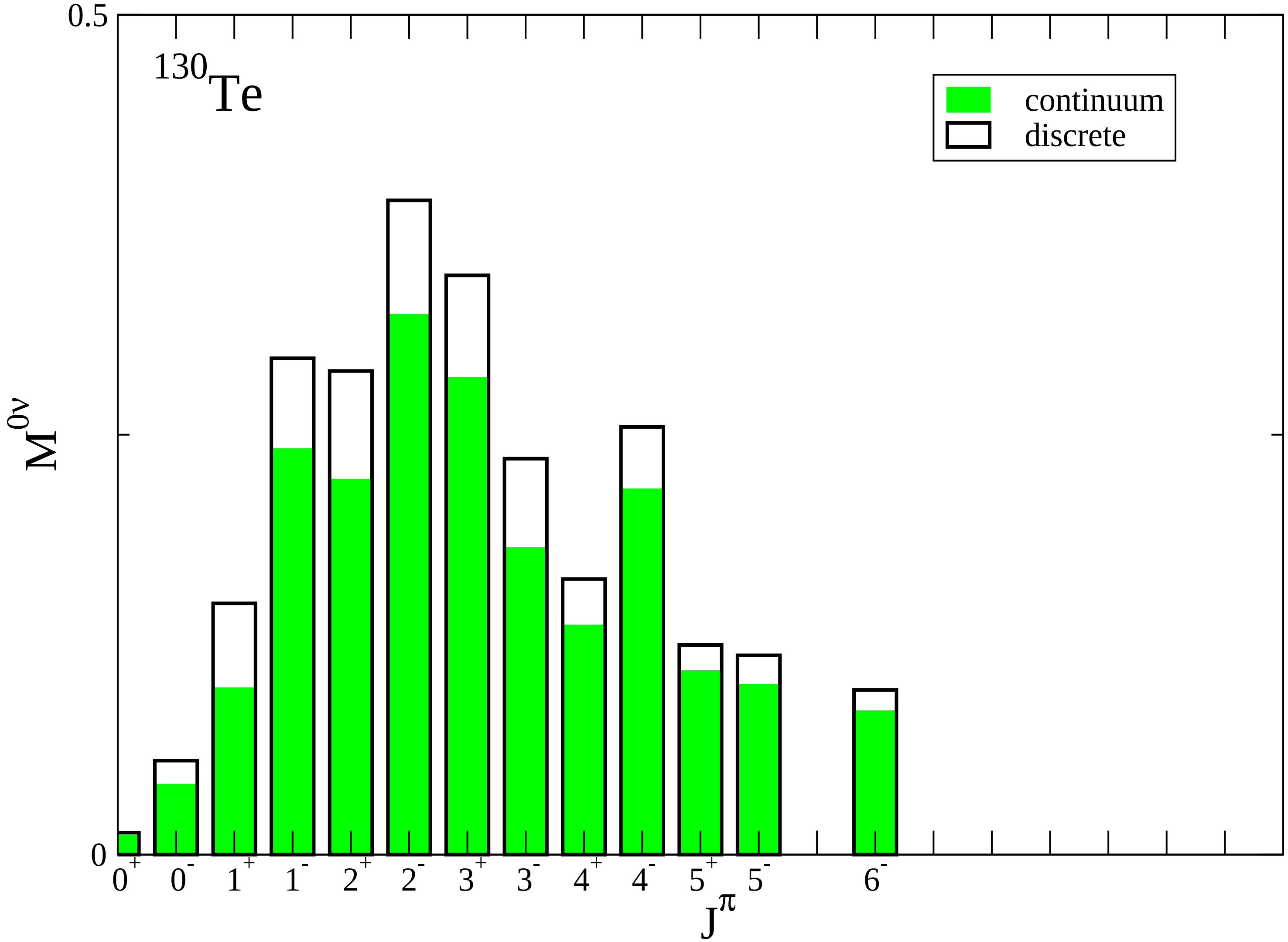
QRPA	f^1_{ph}	g^1_{pp}	$M^{0\nu}$
discrete	0.60	1.20	2.24
continuum	0.65	1.15	1.79

Present calculation of $M^{0\nu}$

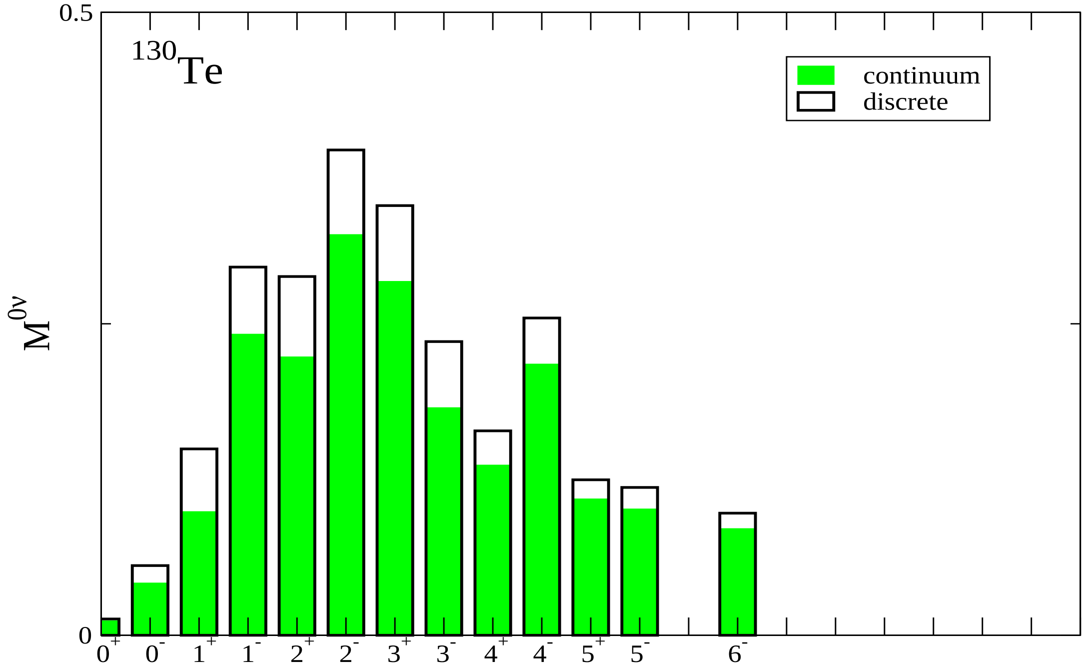
- two-nucleon short-range correlations are included
- the higher order terms of the nucleon current are missing (they reduce $M^{0\nu}$ by about 30%)



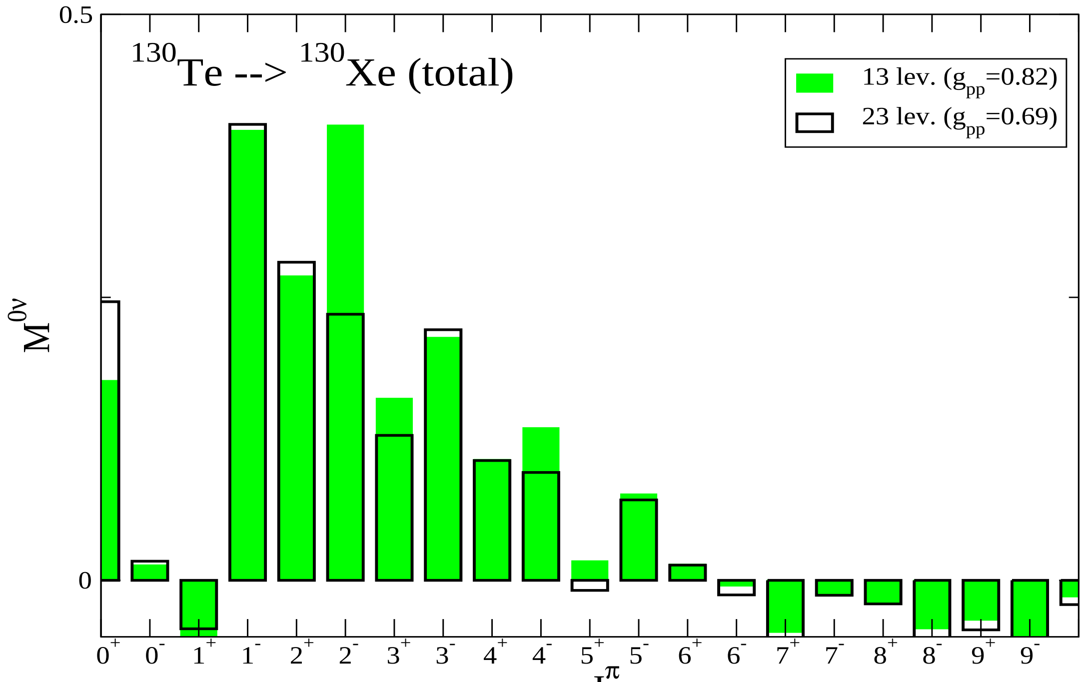
^{130}Te (pairing only)
continuum
discrete
$M_F^{0\nu}$
J^π
0
0.5
1
1.5
2
2.5
3
0^+
1^-
2^+
3^-
4^+
5^-



0.5
0
^{130}Te
continuum
discrete
$M^{0\nu}$
0^+ 0^- 1^+ 1^- 2^+ 2^- 3^+ 3^- 4^+ 4^- 5^+ 5^- 6^-
J^{π}



^{130}Te
continuum
discrete
$M^{0\nu}$
0.5
0
0+ 0- 1+ 1- 2+ 2- 3+ 3- 4+ 4- 5+ 5- 6-



^{130}Te --> ^{130}Xe (total)
13 lev. (g_{pp}=0.82)
23 lev. (g_{pp}=0.69)
$M^{0\nu}$
0.5
0
0+ 0- 1+ 1- 2+ 2- 3+ 3- 4+ 4- 5+ 5- 6+ 6- 7+ 7- 8+ 8- 9+ 9-
J^{π}

Conclusions

- Continuum-QRPA approach to calculation of DBD amplitudes has been formulated

- For ^{130}Te a regular suppression (about 30%) of the $L \geq 2$ multipole contributons to $M^{0\nu}$ has been found

- Total $M^{0\nu}$ for ^{130}Te gets suppressed by about 20%

- Perspective: systematic analysis of other nuclei within cQRPA