Graduiertentag
Kepler Center for Astro and Particle Physics

Neutrino Theory - II: Neutrinos and beyond Standard Model

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Outline

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Dirac versus Majorana mass

construct a Lorentz-invariant mass terms from chiral spinors

- Dirac mass term: two independent chiral 4-spinor fields ψ_L and ψ_R

$$-m\bar{\psi}_R\psi_L + \mathrm{h.c.} = -m\bar{\psi}\psi \quad \text{with} \quad \psi = \psi_L + \psi_R$$

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- Majorana mass term: one independent chiral 4-spinor field ψ_L

$$\frac{1}{2}m\psi_L^T C^{-1}\psi_L + \mathrm{h.c.} = -\frac{1}{2}m\bar{\psi}\psi \quad \text{with} \quad \psi = \psi_L + (\psi_L)^c$$

  with C charge conjugation matrix and $(\psi_L)^c \equiv C\gamma_0^T\psi_L^*$

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  with C charge conjugation matrix and $(\psi_L)^c \equiv C\gamma_0^T\psi_L^*$
  ψ fulfills the Majorana condition $\psi = \psi^c$
  ψ contains annihilation and creation operators $a, a^\dagger \to$ only particles with positive and negative helicity (2 dof)

Lepton number

- Dirac mass term:

$$-m\bar{\psi}_R\psi_L + \mathrm{h.c.} = -m\bar{\psi}\psi \quad \text{with} \quad \psi = \psi_L + \psi_R$$

invariant under a $U(1)$ symmetry $\psi_{L,R} \to e^{i\alpha}\psi_{L,R}$
conserved quantum number (charge, lepton number,...)
$\Rightarrow$ any charged Fermion has to be a Dirac particle

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- Majorana mass term:

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no $U(1)$ symmetry $\Rightarrow$ cannot assign a conserved quantum number (e.g., charge or lepton number) to a Majorana particle
$\Rightarrow$ a Majorana mass term violates lepton number

Dirac mass matrix

Let's consider *n*-generations of Dirac neutrinos:

$$-\bar{\nu}_R \mathcal{M} \nu_L + \mathrm{h.c.} = -\bar{\nu}'_R m \nu'_L + \mathrm{h.c.}$$

where $\nu_{L,R}, \nu'_{L,R}$ are vectors of length n and $\mathcal{M}$ is an arbitrary complex $n \times n$ matrix which can be diagonalized with a bi-unitary transformation:

$$U_R^\dagger \mathcal{M} U_L = m \,.$$

Here m is a diagonal matrix with real and positive entries, U_R, U_L are unitary matrices and

$$\nu_L = U_L \nu'_L \qquad \nu_R = U_R \nu'_R$$

Majorana mass matrix

Let's consider *n*-generations of Majorana neutrinos:

$$\frac{1}{2}\nu_L^T C^{-1} \mathcal{M} \nu_L + \text{h.c.} = \frac{1}{2}\nu'^T_L C^{-1} m \nu'_L + \text{h.c.}$$

where ν_L, ν'_L are vectors of length n and $\mathcal{M}$ is a symmetric complex $n \times n$ matrix:

$$\mathcal{M} = \mathcal{M}^T$$

(follows from anticommutation of fermionic fields and $C^T = -C$).
Such a matrix can be diagonalized by

$$U_L^T \mathcal{M} U_L = m \,,$$

where m is a diagonal matrix with real and positive entries, U_L is a unitary matrix, and

$$\nu_L = U_L \nu'_L$$

Lepton mixing

$$\mathcal{L}_{\mathrm{CC},\ell} = -\frac{g}{\sqrt{2}} W^\rho \, \bar{\ell}_L \gamma_\rho \, \mathbf{U}_{\mathsf{PMNS}} \, \nu'_L - \bar{\ell}_R m^{(\ell)} \ell_L + \mathrm{h.c.}$$

$$\mathcal{L}_{\mathrm{Dirac}} = -\bar{\nu}'_R m \nu'_L + \mathrm{h.c.} \quad \text{or} \quad \mathcal{L}_{\mathrm{Maj}} = \frac{1}{2} \nu'^T_L C^{-1} m \nu'_L + \mathrm{h.c.}$$

$$\mathbf{U}_{\mathsf{PMNS}} \equiv (U_L^{(\ell)})^\dagger U_L$$

Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix

- $U_L^{(\ell)}$ from the diagonalisation of the charged lepton mass matrix
- U_R and $U_R^{(\ell)}$ are unphysical

Lepton mixing

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In processes where only $\mathcal{L}_{\mathrm{CC}}$ (and/or $\mathcal{L}_{\mathrm{NC}}$) is relevant one cannot distinguish between Dirac or Majorana neutrinos

$\Rightarrow$ need a lepton-number violating process

Lepton mixing

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for Dirac neutrinos we can redefine fields as

$$\nu'_L \to e^{i\alpha_\nu} \nu'_L, \; \nu'_R \to e^{i\alpha_\nu} \nu'_R, \; \ell_L \to e^{i\alpha_\ell} \ell_L, \; \ell_R \to e^{i\alpha_\ell} \ell_R,$$

which leads to $U_{\mathrm{PMNS}} \to e^{-i\alpha_\ell} U_{\mathrm{PMNS}} e^{i\alpha_\nu}$. This can be used to eliminate phases on the right and left of U_{PMNS}, only “Dirac phases” remain physical:

$$U_{\mathrm{PMNS}} \to V_{\mathrm{Dirac}}$$

for 2 (3)-flavours V_{Dirac} contains 0 (1) phases

Lepton mixing

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for Majorana neutrinos we can only redefine leptons but not neutrinos:

$$\ell_L \to e^{i\alpha_\ell} \ell_L, \; \ell_R \to e^{i\alpha_\ell} \ell_R \quad \to \quad U_{\mathrm{PMNS}} \to e^{-i\alpha_\ell} U_{\mathrm{PMNS}}$$

cannot absorb phases on the right side of U_{PMNS}

$\Rightarrow (n-1)$ physical Majorana phases

$$U_{\mathrm{PMNS}} \to V_{\mathrm{Dirac}} D_{\mathrm{Maj}} \quad \text{with} \quad D_{\mathrm{Maj}} = \mathrm{diag}(e^{i\alpha_i/2})$$

Oscillations cannot distinguish btw Dirac and Majorana

effective Hamiltonian in matter:

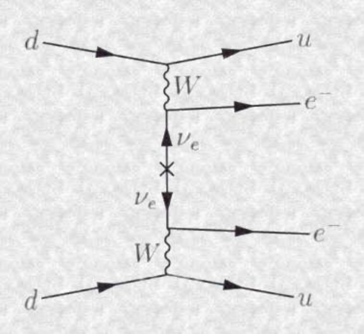
$$
\begin{aligned}
H^{\nu}_{\rm mat} &= U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger + \mathrm{diag}(\sqrt{2} G_F N_e, 0, 0) \\
H^{\bar\nu}_{\rm mat} &= \underbrace{U^* \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^T}_{H_{\rm vac}} - \underbrace{\mathrm{diag}(\sqrt{2} G_F N_e, 0, 0)}_{V_{\rm mat}}
\end{aligned}
$$

$N_e(x)$: electron density along the neutrino path

- oscillations are lepton number conserving
- $U = V_{\rm Dirac} D_{\rm Maj} \Rightarrow$ Majorana phases do not show up in oscillations

Neutrinoless double beta decay

$$(A, Z) \to (A, Z+2) + 2e^-$$



lepton number violating process

$$\Gamma \propto \sum_i U_{ei}^2 m_i$$

depends also on Majorana phases

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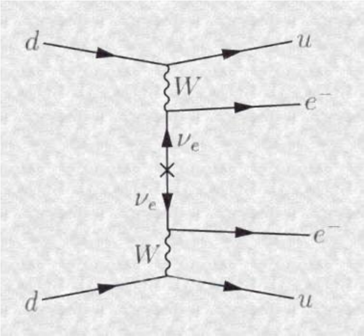

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an observation of neutrinoless DBD implies Majorana nature of neutrinos

Schechter, Valle, 1982; Takasugi, 1984

If neutrinoless DBD is observed, it is not possible to find a symmetry which forbids a Majorana mass term for neutrinos $\Rightarrow$ in a "natural" theory a Majorana mass will be induced at some level.

Outline

Fermion masses in the Standard Model

fermions of one generation:

$$\text{quarks: } Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ u_R,\ d_R \qquad \text{leptons: } L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},\ e_R$$

mass terms from Yukawa coupling to Higgs ϕ

$$\mathcal{L}_Y = -\lambda_d \bar{Q}_L \phi d_R - \lambda_u \bar{Q}_L \tilde{\phi} u_R + \mathrm{h.c.} \qquad -\lambda_e \bar{L}_L \phi e_R + \mathrm{h.c.}$$

$$\text{EWSB} \rightarrow \quad -m_d \bar{d}_L d_R - m_u \bar{u}_L u_R + \mathrm{h.c.} \qquad -m_e \bar{e}_L e_R + \mathrm{h.c.}$$

$$\tilde{\phi} \equiv i\sigma_2 \phi^*,\ m_d = \lambda_d \frac{v}{\sqrt{2}},\ m_u = \lambda_u \frac{v}{\sqrt{2}},\ m_e = \lambda_e \frac{v}{\sqrt{2}},\ \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

No mass term for neutrinos because of absence of ν_R

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No mass term for neutrinos because of absence of ν_R

In the SM neutrinos are massless because. . .

- there are no right-handed neutrinos to form a Dirac mass term, and
- because of the field content and gauge symmetry lepton number [1] is an accidental global symmetry of the SM and therefore no Majorana mass term can be induced.

[1] B-L at the quantum level

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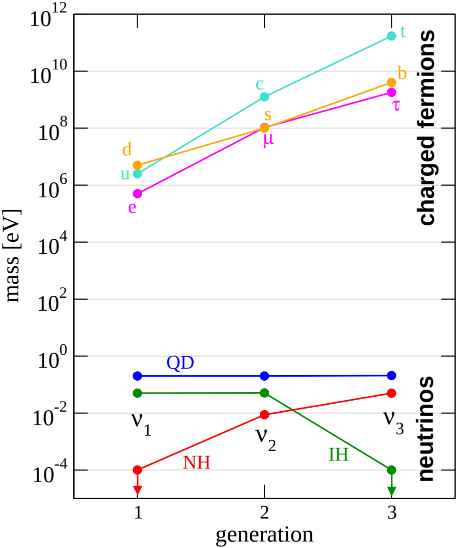
Neutrino mass implies physics beyond the Standard Model

[1] B-L at the quantum level

Why are neutrino masses so small?



Why is lepton mixing large?

Lepton mixing:

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \epsilon \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

Quark mixing:

$$U_{CKM} = \begin{pmatrix} 1 & \epsilon & {\scriptstyle \epsilon} \\ \epsilon & 1 & {\scriptstyle \epsilon} \\ {\scriptstyle \epsilon} & {\scriptstyle \epsilon} & 1 \end{pmatrix}$$

Is there a special pattern in lepton mixing?

example: Tri-bimaximal mixing

Harrison, Perkins, Scott, PLB 2002, hep-ph/0202074

$$\sin^2\theta_{12} = 1/3\,, \quad \sin^2\theta_{23} = 1/2\,, \quad \sin^2\theta_{13} = 0 \qquad \Rightarrow$$

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

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Let's add right-handed neutrinos to the SM

quarks: $Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$, u_R, d_R leptons: $L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$, e_R, N_R

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SM + Dirac neutrinos:

- $\lambda_\nu \lesssim 10^{-11}$ for $m_D \lesssim 1$ eV $(\lambda_e \sim 10^{-6})$
- why is there no Majorana mass term for N_R?
  $\Rightarrow$ have to impose lepton number conservation as additional ingredient of the theory to forbid Majorana mass

Let's allow for lepton number violation

$$\mathcal{L}_Y = -\lambda_e \bar{L}_L \phi e_R - \lambda_\nu \bar{L}_L \tilde{\phi} N_R + \frac{1}{2} N_R^T C^{-1} M_R^* N_R + \mathrm{h.c.}$$

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What is the value of M_R?

We do not know!

There is no guidance from the SM itself because N_R is a gauge singlet
M_R is a new scale in the theory, the scale of BSM physics

The Dirac+Majorana mass matrix

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using $\psi^T C^{-1} = -\overline{\psi^c}\,, \quad \psi^c \equiv C\overline{\psi}^T$

$$\Rightarrow \quad \mathcal{L}_\mathcal{M} = \frac{1}{2} n^T C^{-1} \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} n + \mathrm{h.c.} \quad \text{with} \quad n \equiv \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

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ν_L contains 3 SM neutrino fields, N_R can contain any number r of fields ($r \geq 2$ if this is the only source for neutrino mass, often $r = 3$)

m_D is a general $3 \times r$ complex matrix, M_R is a symmetric $r \times r$ matrix

The Seesaw mechanism

let's assume $m_D \ll M_R$, then the mass matrix $\begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}$ can be approximately block-diagonalized to

$$\begin{pmatrix} m_\nu & 0 \\ 0 & M_R \end{pmatrix} \quad \text{with} \quad m_\nu = -m_D^T M_R^{-1} m_D \sim -\frac{m_D^2}{M_R}$$

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Seesaw:

ν_L are light because N_R are heavy

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$m_D = \lambda v/\sqrt{2}$

- assuming $\lambda \sim 1$ we need $M_R \sim 10^{14}$ GeV for $m_\nu \lesssim 1$ eV
  very high scale - close to $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV
  GUT origin of neutrino mass?

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  GUT origin of neutrino mass?
- m_D could be lower, e.g., $m_D \sim m_e \Rightarrow M_R \sim$ TeV
  e.g., TeV scale L-R symmetric theories
  potentially testable at collider experiments like LHC

Type-II Seesaw

We do not need right-handed neutrinos to give mass to ν_L!

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We do not need right-handed neutrinos to give mass to ν_L!

Let's add a triplet Δ under $SU(2)_L$ to the SM:

$$\mathcal{L}_\Delta = f_{ab}\, L_a^T\, C^{-1}\, i\tau_2 \Delta\, L_b + \text{h.c.}\,,$$

$$\Delta = \begin{pmatrix} H^+/\sqrt{2} & H^{++} \\ H^0 & -H^+/\sqrt{2} \end{pmatrix}$$

The VEV of the neutral component $\langle H^0 \rangle \equiv v_T/\sqrt{2}$ induces a Majorana mass term for the neutrinos:

$$\frac{1}{2}\nu_{La}^T C^{-1} m_{ab}^\nu\, \nu_{Lb} + \text{h.c.} \qquad \text{with} \qquad m_{ab}^\nu = \sqrt{2}\, v_T\, f_{ab}$$

Type-II Seesaw

$$m^{\nu}_{ab} = \sqrt{2}\, v_T\, f_{ab} \lesssim 10^{-10}\,\mathrm{GeV}$$

scalar potential: $$\mathcal{L}_{\mathsf{scalar}}(\phi, \Delta) = -\frac{1}{2} M_\Delta \mathrm{Tr}\Delta^\dagger \Delta + \mu \phi^\dagger \Delta \tilde{\phi} + \ldots$$

μ-term violates lepton number (Δ has $L = -2$)

minimisation of potential: $$v_T \simeq \mu \frac{v^2}{M_\Delta^2}$$

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Type-II seesaw: heavy triplet

$$\mu \sim M_\Delta \sim 10^{14}\,\mathrm{GeV} \qquad \Rightarrow \qquad v_T \sim \frac{v^2}{M_\Delta^2} \sim m^\nu\,,\ f_{ab} \sim \mathcal{O}(1)$$

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triplet at the EW scale $\mathcal{O}$(100 GeV): $M_\Delta \sim v \quad \Rightarrow \quad v_T \sim \mu$

need combination of "small" μ and "small" f_{ab}

The triplet at LHC

$$pp \to Z^*(\gamma^*) \to H^{++}H^{--} \to \ell^+\ell^+\,\ell^-\ell^-$$

doubly charged component of the triplet:

$$\Delta = \begin{pmatrix} H^+/\sqrt{2} & H^{++} \\ H^0 & -H^+/\sqrt{2} \end{pmatrix}$$

very clean signature: two like-sign lepton paris with the same invariant mass and no missing transverse momentum; practically no SM background

The triplet at LHC

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Decays of the triplet:

$$\Gamma(H^{++} \to \ell_a^+ \ell_b^+) = \frac{1}{4\pi(1+\delta_{ab})} \, |f_{ab}|^2 M_\Delta \, ,$$

$\Rightarrow$ proportional to the elements of the neutrino mass matrix!

Type I+II seesaw

assume N_R, Δ_L, Δ_R (e.g., $L-R$ symmetric theories or SO(10) GUTs)

$\langle\Delta_L\rangle$ gives Majorana mass term for ν_L
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Yukawa with Higgs gives Dirac mass term

$$\begin{pmatrix} M_L & m_D^T \\ m_D & M_R \end{pmatrix} \quad \Rightarrow \quad m_\nu = M_L - m_D^T M_R^{-1} m_D$$

assuming $M_L \ll m_D \ll M_R$

R-parity violating SUSY

In SUSY usually conservation of R-parity

$$R \equiv (-1)^{2S+3B+L}$$

is introduced to prevent large B and/or L violation
(fast proton decay, too large neutrino masses)
as a bonus it provides a stable LSP for Dark Matter

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Allow for "tiny" R-parity violation $\Rightarrow$

neutrino mass generation is related to lepton number violating terms in superpotential

can study neutrino properties by observing R-parity violating decays of the LSP (neutralino) at LHC

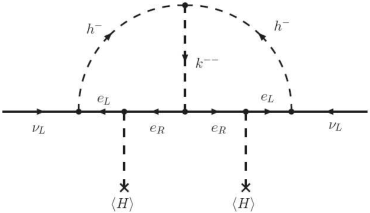
e.g.: Diaz, Dedes, Eboli, Hirsch, Porod, Restrepo, Romao, Valle, ...

Radiative neutrino mass generation

Ex.: Zee-Babu model Zee, 85, 86; Babu 88 add $SU(2)$-singlet scalars: h^+, k^{++}

$$\mathcal{L}_\nu = f_{\alpha\beta} L_\alpha^T C i\sigma_2 L_\beta h^+ + g_{\alpha\beta} \overline{e_{R\alpha}^c} e_{R\beta} k^{++} + \mu h^- h^- k^{++} + \mathrm{h.c.}$$

$$m_\nu \approx \frac{\mu}{48\pi^2 m_k^2} f\, m_\ell\, g^*\, m_\ell\, f^T$$



good prospects to see doubly-charged scalar at LHC → like-sign lepton events if k^{++} is within reach for LHC, tight constrains by perturbativity requirements and bounds from LFV Babu, Macesanu, 02; Aristizabal, Hirsch, 06; Nebot et al., 07, Ohlsson, TS, Zhang, 09

The Weinberg operator

Assume there is new physics at a high scale Λ. It will manifest itself by non-renormalizable operators suppressed by powers of Λ.

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In the 1930's Fermi did not know about W and Z bosons, but he could write down a non-renormalizable dimension-6 operator to describe beta decay:

$$\frac{g^2}{\Lambda^2}(\bar{e}\gamma_\mu\nu)(\bar{p}\gamma^\mu n)$$

- Fermi knew about charge conservation $\longrightarrow$ his operator is invariant under $U(1)_{\text{em}}$
- Today we know that $\Lambda \simeq m_W$, and we know the UV completion of Fermi's operator, i.e. the electro-weak theory of the SM.

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Weinberg 1979: there is only one dim-5 operator consistent with the gauge symmetry of the SM, and this operator will lead to a Majorana mass term for neutrinos after EWSB:

$$Y^2 \frac{L^T \tilde{\phi}^* \, \tilde{\phi}^\dagger L}{\Lambda} \quad \longrightarrow \quad m_\nu \sim Y^2 \frac{v^2}{\Lambda}$$

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3 tree-level realizations of the Weinberg operator:

- Type I: fermionic singlet (right-handed neutrinos)
- Type II: scalar triplet
- Type III: fermionic triplet

High-scale versus low-scale seesaw

can obtain small neutrino masses by making Λ very large or Y very small (or both)

- High scale seesaw: $\Lambda \sim 10^{14}$ GeV, $Y \sim 1$
  - "natural" explanation of small neutrino masses
  - Leptogenesis
  - very hard to test experimentally

- Low scale seesaw: $\Lambda \sim$ TeV
  - link neutrino mass generation to new physics testable at colliders
  - observable signatures in searches for LFV
    $\mu \to e\gamma, \tau \to \mu\gamma, \mu \to eee, \ldots$

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Outline

The baryon asymmetry

the asymmetry between baryons and antibaryons in the early Universe was $\eta_B \sim 10^{-10}$:

baryons:	$+\ 10\,000\,000\,001$
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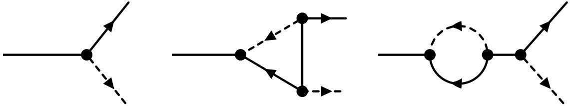
$\Rightarrow$ **requires physics beyond the SM**

Leptogenesis

M. Fukugita, T. Yanagida, Phy. Lett. B174, 45 (1986)

assume type-I seesaw with heavy ($\sim 10^{10}$ GeV) right-handed neutrinos N

- out of equilibrium decay of $N \to \phi\ell$
- CP asymmetry in N decays: $\Gamma(N \to \phi^+\ell^-) \neq \Gamma(N \to \phi^-\ell^+)$
  due to tree- and loop-level interference



  net-lepton number L is generated

- L is transformed to baryon number by non-perturbative $B - L$ conserving (but $B + L$ violating) sphaleron processes in the SM

Leptogenesis

(+) elegant mechanism to explain baryon asymmetry

(+) links neutrino physics to our existence

(+) many versions (with or without lepton number violation, for all types of seesaw, Dirac Leptogensis, TeV-scale Leptogenesis, . . .)

(−) in general can neither be tested nor excluded by low-energy experiments at best we can obtain "circumstantial evidence":

- observe neutrinoless double beta decay (Majorana nature),
- observe CP violation in oscillations,

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Conclusion

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- More potentially exciting discoveries are ahead.
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Thank you for your attention!