Alfred Rieckers
Kurt Bräuer

# Einladung zur Mathematik

*Eine mathematische Einführung und Begleitung zum Studium der Physik und Informatik, Logos Verlag 2002*

*Lösungen zu den Übungsaufgaben*

(Fehlerhinweise bitte an kurt.braeuer@uni-tuebingen.de)

# Kapitel 7

**Übungsbeispiel 7.3**

Partielle Integration

$$\int \underset{u}{x} \underset{v'}{\sin x}\, dx = -x\cos x + \int \cos x dx = -x\cos x + \sin x$$

$$\int \underset{u}{x} \underset{v'}{\cos x}\, dx = +x\sin x - \int \sin x dx = +x\sin x + \cos x$$

$$\int \underset{u}{x^2} \underset{v'}{\cos x}\, dx = +x^2 \sin x - 2\underbrace{\int x\sin x dx}_{-x\cos x+\sin x} = x^2\sin x - 2\left(-x\cos x + \sin x\right) = x^2\sin x + 2x\cos x - 2\sin x$$

$$\int \underset{u}{x^n} \underset{v'}{e^x}\, dx = x^n e^x - n\int x^{n-1} e^x dx$$

$$\int \underset{u}{\sin x} \underset{v'}{\cos x}\, dx = \sin^2 x - \int \cos x \sin x dx \quad \Rightarrow \quad \int \sin x\cos x dx = \frac{\sin^2 x}{2}$$

$$\int \sin^n x dx = \int \underbrace{\sin^{n-1} x}_{u} \underset{v'}{\sin x}\, dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\cos^2 x dx$$

$$\int \underset{u'}{x} \underset{v}{\ln x}\, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \frac{1}{x} = \frac{1}{2}x^2\ln x - \frac{1}{2}\int x = \frac{1}{4}x^2\left(2\ln x - 1\right)$$

Substitution

$$\int \sin^3 x \underbrace{\cos x dx}_{=d\sin x} = \int \sin^3 x d\sin x = \frac{1}{4}\sin^4 x$$

$$\int \sin^3 x dx = \int \left(1-\cos^2 x\right) \underbrace{\sin x dx}_{=-d\cos x} = -\cos x + \frac{1}{3}\cos^3 x$$

$$\int x e^{x^2} dx = \int e^{x^2} \underbrace{x dx}_{\frac{1}{2}dx^2} = \frac{1}{2}e^{x^2}$$

$$\int x\sqrt{x^2+a^2}\,dx = \int \sqrt{x^2+a^2} \underbrace{x dx}_{\frac{1}{2}d\left(x^2+a^2\right)} = \frac{1}{3}\left(x^2+a^2\right)^{3/2}$$

## Übungsaufgaben

$$\int \frac{dx}{\left(x-a\right)^1} = \int \frac{d\left(x-1\right)}{\left(x-a\right)^1} = \ln\left(x-a\right)$$

$$\int \frac{dx}{\left(x-a\right)^n} = \int \frac{d\left(x-1\right)}{\left(x-a\right)^n} = \frac{1}{n-1}\frac{1}{\left(x-a\right)^{n-1}}$$