# Time-energy uncertainty relation and neutrino oscillations

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Basic facts from today's data

I. Neutrinos are massive and oscillate

II. The number of massive neutrinos is equal to the number of flavor neutrinos (three)

III. Three-neutrino mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\, \nu_{iL}, \quad l = e, \mu, \tau$$

IV. Standard transition probability

$$P(\nu_l \to \nu_{l'}) = |\sum_{i=1}^{3} U_{l'i} e^{-i \Delta m^2_{1i} \frac{L}{2E}} U^*_{li}|^2$$

## V. Two parameters are small

$$\frac{\Delta m_{12}}{\Delta m_{23}} \simeq 3 \cdot 10^{-2}, \quad \sin^2\theta_{13} \leq 5 \cdot 10^{-2}$$

In the leading approximation

At $m^2_{23}\frac{L}{2E} \gtrsim 1$ (atmospheric and LBL experiments)

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{1}{2}\sin^2 2\theta_{23}(1 - \cos\Delta m^2_{23}\frac{L}{2E})$$

$$(\nu_\mu \leftrightarrows \nu_\tau)$$

At $m^2_{12}\frac{L}{2E} \gtrsim 1$ (KamLAND)

$$P(\bar\nu_e \to \bar\nu_e) = 1 - \frac{1}{2}\sin^2 2\theta_{12}(1 - \cos\Delta m^2_{12}\frac{L}{2E})$$

$$(\bar\nu_e \leftrightarrows \bar\nu_{\mu,\tau})$$

Solar neutrino data are described by two-neutrino $\nu_e$ survival probability in matter which depends on $\Delta m^2_{12}$ and $\sin^2\theta_{12}$

Decoupled two-neutrino oscillations in two regions of neutrino mass-squared differences were observed

Next step : measuring of the value of the parameter $\sin^2 2\theta_{13}$ and search for beyond the leading approximation effects of three neutrino mixing (CP violation, character of neutrino mass spectrum)

Require high-precision, challenging experiments and exact theory of neutrino oscillations

Because neutrino masses are small and $\frac{m_i^2}{E^2} \lll 1$ basically different assumptions give the same standard expression for the transition probability

The quantum origin of neutrino oscillations can not be tested in standard neutrino oscillation experiment. Special experiments are necessary

## Two major assumptions

I. Evolution of neutrino states in time, neutrino oscillations is nonstationary phenomenon

$$P(\nu_l \to \nu_{l'}) = |\sum_{i=1}^{3} U_{l'i} e^{-iE_i t} U_{li}^{*}|^2$$

$t$ is time interval between neutrino production and detection

$(E_i - E_1)t = \Delta m^2_{1i} \frac{L}{2E}$ (standard phase difference)

Assumptions

- same momenta $p_i = p_k = p$
- $t = L$ ( ultra relativistic neutrinos)

## II. Evolution of neutrino states in time and space

$$P(\nu_l \to \nu_{l'}) = |\sum_{i=1}^{3} U_{l'i} e^{-iE_i t + ip_i L} U_{li}^*|^2$$

$E_i = \sqrt{p_i^2 + m_i^2}$, $t$ is time interval and $L$ is the distance between neutrino production and detection points

Phase difference

$$\phi_{1i} = (E_i - E_1)t - (p_i - p_1)x$$

A. Same momenta, $\phi_{1i} = \Delta m_{1i}^2 \frac{L}{2E}$ (previous assumption $t = L$)

B. Same energies (stationary case) $\phi_{1i} = \Delta m_{1i}^2 \frac{L}{2E}$

C. Different momenta, different energies

$$\phi_{1i} = \frac{E_i^2 - E_1^2}{E_i` - E_1} t + (p_i - p_1)x = \Delta m_{1i}^2 \frac{L}{2E} + O(\frac{m_i^4}{E^4})$$

(assumption $t = L + O(\frac{m_i^2}{E^2})$)

In all cases we come to the same standard phase differences (oscillation phases)

What picture is correct?

General question

Can neutrino oscillations be observed in the stationary processes(same neutrino energies)?

For non stationary processes time-energy uncertainty relation

$$\Delta E \Delta t \geq \frac{1}{2}$$

takes place

Time-energy uncertainty relation gives a connection between uncertainty of energy $\Delta E$ and an interval of time $\Delta t$ during which a state of a system changes

There were a lot of discussions and many derivations of this relation (see, for example, P.Bush quant-ph/0105049)

A general method of derivation of the time-energy uncertainty relation was proposed by Mandelstam and Tamm

We will use their method

For any hermitian operators $A$ and $B$ and any state $|\Psi\rangle$ the following inequality (which can be easily obtained from Couchy-Schwarz inequality) is valid

$$\Delta A \Delta B \geq \frac{1}{2} |\langle\Psi|[A, B]|\Psi\rangle|$$

$$\Delta A = \sqrt{\langle\Psi|(A - \overline{A})^2 \Psi\rangle}$$

Thus, a product of uncertainties of two operators is determined the matrix element of their commutator

This inequality is basis for uncertainty relations For example, from $[p, q] = \frac{1}{i}$ we have $\Delta p \Delta q \geq \frac{1}{2}$ etc

Heisenberg uncertainty relations are valid for operators which do not commute and their form is determined by the value of the commutator

To derive time-energy uncertainty relation Mandelstam and Tamm used the same inequality. However, time in the quantum theory is not operator. It is a parameter which describes evolution of the system

Commutator of any Heisenberg operator $O_H(t)$ with the total Hamiltonian $H$ is given

$$[O_H(t), H] = i\frac{dO_H(t)}{dt}$$

Evolution is determined by Hamiltonian

Using general inequality we have

$$\Delta E \Delta O_H(t) \geq |\frac{1}{2}\frac{d}{dt}{}_H\langle\Psi|O_H(t)|\Psi\rangle_H|$$

$$O_H(t) = e^{iHt}Oe^{-iHt}$$

$O$ is corresponding operator in the Schrodinger representation

In order to derive time-energy relation for neutrino oscillation we choose projection operator

$$O = |\nu_l\rangle\ \langle\nu_l|$$

$|\nu_l\rangle$ is the state of flavor neutrino $\nu_l$

$$\langle\nu_{l'}|\nu_l\rangle = \delta_{l'l}$$

With this choice $O_H^2(t) = O_H(t)$

$${}_H\langle\Psi|O_H(t)|\Psi\rangle_H = P_{\nu_l\to\nu_l}(t) \quad \Delta O_H(t) = \sqrt{P_{\nu_l\to\nu_l}(t) - P^2_{\nu_l\to\nu_l}(t)}$$

$P_{\nu_l\to\nu_l}(0)$ is $\nu_l$ survival probability (we assumed that initial neutrino state is $|\nu_l\rangle$)

$$\Delta E \geq \frac{1}{2}\ \frac{|\frac{d}{dt}P_{\nu_l\to\nu_l}(t)|}{\sqrt{P_{\nu_l\to\nu_l}(t) - P^2_{\nu_l\to\nu_l}(t)}}$$

Right-hand side characterize a change of the survival probability at the moment $t$. In the time-energy uncertainty relation enters time interval $\Delta t$ during which a state is changed

We will perform an integration over a time interval during which survival probability is significantly changed

We choose

$$\Delta t = t_2 - t_1, \quad t_2 = t_{\rm min}, \quad t_1 = 0$$

$t_{\rm min}$ is the time of the first minimum of the survival probability

After integration

$$\Delta E \; \Delta t \geq \frac{1}{2} \; \left(\frac{\pi}{2} - \arcsin(2 \; P^{\rm min}_{\nu_l \to \nu_l} - 1)\right)$$

$$P^{\rm min}_{\nu_l \to \nu_l} = P_{\nu_l \to \nu_l}(t_{\rm min})$$

Consider $\nu_\mu \to \nu_\mu$ transitions driven by atmospheric-LBL $\Delta m^2_{23}$

In this case

$$t^{(23)}_{\rm min} = 2\pi \; \frac{E}{\Delta m^2_{23}} = \frac{1}{2} t_{\rm osc}, \quad P_{\nu_\mu \to \nu_\mu}(t_{\rm min}) \simeq 0$$

Time-energy uncertainty relation takes the form

$$\Delta E \; \Delta t \geq \pi$$

From time-energy uncertainty relation

$\Delta E \geq \frac{1}{2}\frac{\Delta m^2_{23}}{E}, \quad \Delta E \gtrsim 10^{-12}\mathrm{eV}$

Easily satisfied in experiments without any constraints on production mechanism

For ultra relativistic neutrinos

$$\Delta t \simeq \Delta x$$

$\Delta x \equiv L$ is the distance between neutrino source and detector, $\Delta t$ is time interval between production and detection of neutrinos

This relation was checked in K2K and MINOS experiments. In the K2K experiment neutrinos are produced in 1.1 $\mu s$ spills and time of neutrino detection was measured.

$$-0.2 \leq \left(\Delta t - \frac{\Delta x}{c}\right) \leq 1.3\ \mu s, \quad \Delta t \simeq 0.8 \cdot 10^3 \mu s$$

Let us consider

$\bar{\nu}_e \to \bar{\nu}_e$

driven by $\Delta m^2_{23}$

In this case $P_{\bar{\nu}_e \to \bar{\nu}_e}(t^{(23)}_{\mathrm{min}}) = 1 - \sin^2 2\theta_{13}$
$\sin^2 2\theta_{13} \lesssim 2 \cdot 10^{-1}$ (CHOOZ)

Time-energy uncertainty relation

$$\Delta E \; \Delta t \geq \; \sin 2\theta_{13}$$

Weaker requirement on $\Delta E$

It was proposed by Raghavan an experiment on the detection of the tritium $\bar{\nu}_e$ with energy $\simeq$ 18.6 keV in the recoilless (Mossbauer) transitions

$$^{3}\mathrm{H} \to ^{3}\mathrm{He} + \bar{\nu}_{\mathrm{e}}, \quad \bar{\nu}_{\mathrm{e}} + ^{3}\mathrm{He} \to ^{3}\mathrm{H}$$

Oscillation length in such experiment $L_{\mathrm{osc}}^{(23)} \simeq 18.6\ m$

It was estimated by Raghavan that $(\Delta E)_R \simeq 8.4 \cdot 10^{-12}$ eV and $\sigma_R \simeq 3 \cdot 10^{-33} \mathrm{cm}^2$

(For much more pessimistic estimates see W. Potzel ArXiv: 0810.2170)

If feasible it would be possible to to measure the parameter $\sin^2\theta_{13}$ in the experiment with baseline about 10m etc.

## Mossbauer neutrino experiment from the point of view of the time-energy uncertainty relation

$\Delta E$ which drives neutrino transition during the time $t^{(23)}_{\mathrm{min}}$ must satisfy the inequality

$$\Delta E \geq \frac{1}{2\pi} \sin 2\theta_{13} \frac{\Delta m^2_{23}}{E}$$

If $\sin 2\theta_{13} \neq 0$ a constraint on $\Delta E$

$\sin^2 2\theta_{13} = 2 \cdot 10^{-1}$ (CHOOZ bound)

$\Delta E \geq 0.9 \cdot 10^{-8}$ eV

$\sin^2 2\theta_{13} = \cdot 10^{-2}$ (T2K, Daya Bay,...)

$\Delta E \geq 2.2 \cdot 10^{-9}$ eV

Estimated uncertainty $(\Delta E)_R \simeq 8.4 \cdot 10^{-12}$ eV does not satisfy these inequality

Time-energy uncertainty relation (Mandelstam-Tamm) is not satisfied in Mossbauer neutrino experiment if $\bar{\nu}_e$ survival probability is given by the standard expression

Opposite conclusions (A. Akhmedov, J. Kopp, M. Lindner, A. Cohen, S. Glashow, Z. Ligeti) are based on space-time evolution of neutrino states

CONCLUSION

Mossbauer neutrino experiment is a new type of neutrino experiment (fixed neutrino energy)

It would be very important if such an experiment could be performed