# “Darmstadt oscillations” and related problems in physics of massive neutrinos

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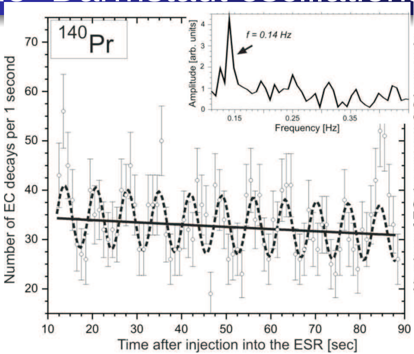
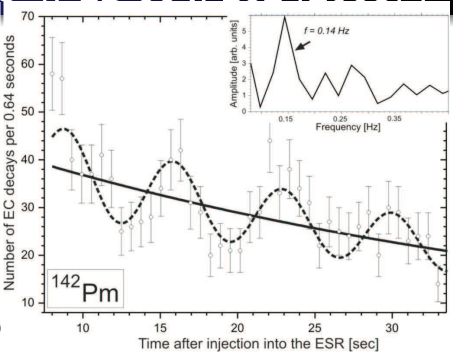
November 18, 2008 / Trento

# Content

# The “Darmstadt oscillations” (PLB **664**, 162 (2008) )




$^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + \nu_e$ $\quad$ $^{142}\mathrm{Pm}^{60+} \to {}^{142}\mathrm{Nd}^{60+} + \nu_e$

# Periodic *EC*–decay rates - the “Darmstadt oscillations”

$$^{140}\mathrm{Pr}^{58+} \longrightarrow {}^{140}\mathrm{Ce}^{58+} + \nu_e \qquad {}^{142}\mathrm{Pm}^{60+} \longrightarrow {}^{142}\mathrm{Nd}^{60+} + \nu_e$$

$$\frac{dN_d^{EC}(t)}{dt} = \lambda_{EC}\, N_m(t) \quad \longrightarrow \quad \frac{dN_d^{EC}(t)}{dt} = \lambda_{EC}(t)\, N_m(t)$$

$$\frac{\lambda_{EC}(t)}{\lambda_{EC}} = 1 + a_{EC}\cos(\omega_{EC} t + \phi_{EC})$$

$$T_{EC} = \frac{2\pi}{\omega_{EC}} \simeq 7\,\mathrm{s} \qquad a_{EC} \simeq 0.20$$

# Properties of nuclei and H–like ions in *EC*–decays

- Nuclei ${}^{140}\mathrm{Pr}^{59+}$ and ${}^{142}\mathrm{Pm}^{61+}$ are the states with $J^P = 1^+$
- Nuclei ${}^{140}\mathrm{Ce}^{58+}$ and ${}^{140}\mathrm{Nd}^{60+}$ are the states with $J^P = 0^+$
- H–like ions ${}^{140}\mathrm{Pr}^{58+}$ and ${}^{142}\mathrm{Pr}^{60+}$: hyperfine states $(1s)_{F=\frac{1}{2}}$ and $(1s)_{F=\frac{3}{2}}$
- Energy level difference: $\Delta E = E_{F=\frac{1}{2}} - E_{F=\frac{3}{2}} \sim -1\,\mathrm{eV}$
- Decay channel of $(1s)_{F=\frac{3}{2}}$: $(1s)_{F=\frac{3}{2}} \to (1s)_{F=\frac{1}{2}} + \gamma$
- Lifetime of $(1s)_{F=\frac{3}{2}}$ state: $\tau_{(1s)_{F=\frac{3}{2}}} \sim 10^{-2}\,\mathrm{s}$
- Selection rule: $\Delta J^P = 1^+$ - Gamow–Teller transition (Konopinski, 1966)

# Decay rates of H-like ${}^{140}\mathrm{Pr}^{58+}$ and He-like ${}^{140}\mathrm{Pr}^{57+}$ ions: GSI, PRL **99**, 262501 (2007))

$$
\begin{aligned}
{}^{140}\mathrm{Pr}^{58+} &\to {}^{140}\mathrm{Ce}^{58+} + \nu_e \to \lambda^{(\mathrm{H})}_{EC} = 0.00219(6)\,\mathrm{s}^{-1},\\
{}^{140}\mathrm{Pr}^{58+} &\to {}^{140}\mathrm{Ce}^{57+} + e^+ + \nu_e \to \lambda^{(\mathrm{H})}_{\beta^+} = 0.00161(10)\,\mathrm{s}^{-1},\\
{}^{140}\mathrm{Pr}^{57+} &\to {}^{140}\mathrm{Ce}^{57+} + \nu_e \to \lambda^{(\mathrm{He})}_{EC} = 0.00147(7)\,\mathrm{s}^{-1}\\
{}^{140}\mathrm{Pr}^{57+} &\to {}^{140}\mathrm{Ce}^{56+} + e^+ + \nu_e \to \lambda^{(\mathrm{He})}_{\beta^+} = 0.00154(11)\,\mathrm{s}^{-1}
\end{aligned}
$$

$$
R^{(\mathrm{H/H})}_{EC/\beta^+} = 1.36(9) \quad R^{(\mathrm{He/He})}_{EC/\beta^+} = 0.96(8) \quad R^{(\mathrm{H/He})}_{EC/EC} = 1.49(8)
$$

$$
\tau = \frac{1}{\lambda}:\ \tau^{(\mathrm{H})}_{EC} = 457\,\mathrm{s}\,,\ \tau^{(\mathrm{H})}_{\beta^+} = 621\,\mathrm{s}\,,\ \tau^{(\mathrm{He})}_{EC} = 680\,\mathrm{s}\,,\ \tau^{(\mathrm{He})}_{\beta^+} = 650\,\mathrm{s}
$$

# First–forbidden continuum and bound state $\beta^-$ decay rates of bare $^{207}\mathrm{Tl}^{81+}$ and $^{205}\mathrm{Hg}^{80+}$ ions

$$^{207}\mathrm{Tl}^{81+} \to {}^{207}\mathrm{Pb}^{82+} + e^- + \tilde{\nu}_e \quad , \quad {}^{207}\mathrm{Tl}^{81+} \to {}^{207}\mathrm{Pb}^{81+} + \tilde{\nu}_e$$

$$^{205}\mathrm{Hg}^{80+} \to {}^{205}\mathrm{Tl}^{81+} + e^- + \tilde{\nu}_e \quad , \quad {}^{205}\mathrm{Hg}^{80+} \to {}^{205}\mathrm{Tl}^{80+} + \tilde{\nu}_e,$$

**Experimental data: (GSI, PRL 95, 052501 (2004)**

$$R^{\exp}_{b/c}\Big|_{^{207}\mathrm{Tl}^{81+}} = 0.188(18)$$

**Experimental data: (GSI, to be published)**

$$R^{\exp}_{b/c}\Big|_{^{205}\mathrm{Hg}^{80+}} = 0.20(2)$$

# Properties of bare and H–like ions in $\beta^-$ decays of bare $^{207}\mathrm{Tl}^{81+}$ and $^{205}\mathrm{Hg}^{80+}$

- Nuclei $^{207}\mathrm{Pb}^{82+}$ and $^{205}\mathrm{Hg}^{80+}$ are the states with $J^P = \frac{1}{2}^-$
- Nuclei $^{207}\mathrm{Tl}^{81+}$ and $^{205}\mathrm{Tl}^{81+}$ are the states with $J^P = \frac{1}{2}^+$
- Hyperfine states of H–like $^{207}\mathrm{Pb}^{81+}$ and $^{205}\mathrm{Tl}^{80+}$ ions : $(ns)_{F=0}$ and $(ns)_{F=1}$: $\Delta E_{ns} = E_{(ns)_{F=0}} - E_{(ns)_{F=1}}$
- Shabaev (1994): $\Delta E^{\mathrm{th}}_{1s} = -3.275\,\mathrm{eV}(\Delta E^{\mathrm{exp}}_{1s} = -3.244\,\mathrm{eV})$

$$\Delta E^{\mathrm{th}}_{ns} = \Delta E_{1s}\frac{2(n+\gamma)+\sqrt{(n+\gamma)^2-\gamma(2+\gamma)}}{(3+2\gamma)[(n+\gamma)^2-\gamma(2+\gamma)]^2}$$

- Selection rule: $\Delta J^P = 0^-$ - first–forbidden $\beta$–decays (Konopinski, 1966)

# Theoretical analysis of weak decays of $^{140}\mathrm{Pr}^{58+}$ and $^{140}\mathrm{Pr}^{57+}$ ions (PRC **78**, 025503 (2008))

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\, [\bar{\psi}_n(x)\gamma^{\mu}(1 - g_A\gamma^5)\, \psi_p(x)][\bar{\psi}_{\nu_e}(x)\gamma_{\mu}(1 - \gamma^5)\psi_e(x)]$$

with a neutrino as a massless elementary Dirac particle

$$\begin{aligned}
\lambda^{(\mathrm{H})}_{EC} &= \frac{1}{2F+1}\,\frac{3}{2}|\mathcal{M}_{\mathrm{GT}}|^2|\langle\psi^{(Z)}_{1s}\rangle|^2\frac{Q^2_{\mathrm{H}}}{\pi}\\
\lambda^{(\mathrm{H})}_{\beta^+} &= \frac{2}{2F+1}\,\frac{|\mathcal{M}_{\mathrm{GT}}|^2}{4\pi^3} f(Q^{\mathrm{H}}_{\beta^+}, Z-1)\\
\lambda^{(\mathrm{He})}_{EC} &= \frac{1}{2I+1}\,\frac{3}{2}|\mathcal{M}_{\mathrm{GT}}|^2|\langle\psi^{(\mathrm{Z}-1)}_{1s}\psi^{(\mathrm{Z})}_{(1s)^2}\rangle|^2\frac{Q^2_{\mathrm{He}}}{\pi}\\
\lambda^{(\mathrm{He})}_{\beta^+} &= \frac{3}{2I+1}\,\frac{|\mathcal{M}_{\mathrm{GT}}|^2}{4\pi^3} f(Q^{\mathrm{He}}_{\beta^+}, Z-1)
\end{aligned}$$

# Theoretical analysis of weak decays of $^{140}\mathrm{Pr}^{58+}$ and $^{140}\mathrm{Pr}^{57+}$ ions (PRC **78**, 025503 (2008))

$$\mathcal{M}_{\rm GT} = -2g_A G_F V_{ud}\int d^3x\,\Psi^*_d(r)\Psi_m(r) = -2g_A G_F V_{ud}\int d^3x\,\rho_{WS}(r)$$

$$\langle\psi^{(Z)}_{1s}\rangle = \frac{\int d^3x\,\psi^{(Z)}_{1s}(\vec{r}\,)\,\rho_{WS}(r)}{\int d^3x\,\rho_{WS}(r)}$$

$$f(Q_{\beta^+},Z) = \int^{Q_{\beta^+}-m_e}_{m_e}(Q_{\beta^+}-m_e-E_+)^2\sqrt{E^2_+ - m^2_e}\,F(Z,E_+)\,E_+ dE_+$$

$$F(Z,E_+) = \Big(1+\frac{1}{2}\gamma\Big)\frac{4(2Rp_+)^{2\gamma}}{\Gamma^2(3+2\gamma)}\,e^{-\pi Z\alpha E_+/p_+}\Big|\Gamma\Big(1+\gamma-i\,\frac{\alpha Z E_+}{p_+}\Big)\Big|^2$$

$$p_+ = \sqrt{E^2_+ - m^2_e}$$

# Theoretical analysis of weak decays of $^{140}\mathrm{Pr}^{58+}$ and $^{140}\mathrm{Pr}^{57+}$ ions (PRC **78**, 025503 (2008))

$$R^{(\mathrm{H}),\mathrm{th}}_{EC/\beta^+} = \frac{3\pi^2 Q^2_{\mathrm{H}} |\langle\psi^{(Z)}_{1s}\rangle|^2}{f(Q_{\beta^+}, Z-1)} = 1.40(4)\ \Big[1.36(9)\Big]^{\mathrm{exp}}$$

$$R^{(\mathrm{He}),\mathrm{th}}_{EC/\beta^+} = \frac{2\pi^2 Q^2_{\mathrm{He}} |\langle\psi^{(Z-1)}_{1s}\psi^{(Z)}_{(1s)^2}\rangle|^2}{f(Q_{\beta^+}, Z-1)} = 0.94(3)\ \Big[0.96(8)\Big]^{\mathrm{exp}}$$

$$R^{(\mathrm{H/He}),\mathrm{th}}_{EC/EC} = \frac{2I+1}{2F+1}\,\frac{|\langle\psi^{(Z)}_{1s}\rangle|^2}{|\langle\psi^{(Z-1)}_{1s}\psi^{(Z)}_{(1s)^2}\rangle|^2}\,\frac{Q^2_{\mathrm{H}}}{Q^2_{\mathrm{He}}} = 1.50(4)\ \Big[1.49(8)\Big]^{\mathrm{exp}}$$

# Theoretical analysis of $\beta^-$ decay rates of bare $^{207}\mathrm{Tl}^{81+}$ and $^{205}\mathrm{Hg}^{80+}$ ions: 0810.2167 [nucl-th]

$$\lambda_{\beta_b^-} \propto 2\sum_{n=1}^{\infty} |\langle\psi_{ns}^{(Z+1)}\rangle|^2\,\frac{Q_{ns}^2}{\pi} \qquad \lambda_{\beta_c^-} \propto \frac{1}{\pi^3} f(Q_{\beta_c^-}, Z+1)$$

$$R_{b/c} = \sum_{n=1}^{\infty} \frac{2\pi^2 Q_{\mathrm{ns}}^2 |\langle\psi_{ns}^{(Z+1)}\rangle|^2}{f(Q_{\beta_c^-}, Z+1)}$$

$$R_{b/c}^{\mathrm{th}}\Big|_{^{207}\mathrm{Tl}^{81+}} = 0.190 \quad [0.188(18)]^{\mathrm{exp}}$$

$$R_{b/c}^{\mathrm{th}}\Big|_{^{205}\mathrm{Hg}^{80+}} = 0.161 \quad [0.20(2)]^{\mathrm{exp}}$$

# Massive neutrinos and weak decays of $^{140}\mathrm{Pr}^{58+}$ and $^{140}\mathrm{Pr}^{57+}$ ions

**Weak interaction Hamilton density operator**

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\,[\bar{\psi}_n(x)\gamma^{\mu}(1 - g_A\gamma^5)\,\psi_p(x)]$$

$$\times \sum_{j=1,2,3} U_{ej}\,[\bar{\psi}_{\nu_j}(x)\gamma_{\mu}(1 - \gamma^5)\psi_e(x)]$$

$$U_{e1} = \cos\theta_{12}\cos\theta_{13}\;,\;U_{e2} = \sin\theta_{12}\cos\theta_{13}\;,\;U_{e3} = \sin\theta_{13}$$

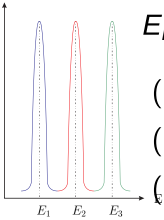
**Solar neutrino data** : $\theta^{\rm exp}_{12} \simeq 34^0 \quad \theta^{\rm exp}_{13} \simeq 0$

# *EC*–decay rates of $^{140}\mathrm{Pr}^{58+}$ and $^{142}\mathrm{Pm}^{60+}$ ions and energy spectrum of neutrino mass–eigenstates

$$\lambda^{(\mathrm{H})}_{EC} = \sum_j |U_{ej}|^2 \lambda^{(\mathrm{H})}_{EC}(m_j) \stackrel{m_j \to 0}{\longrightarrow} \frac{1}{2F+1}\,\frac{3}{2}|\mathcal{M}_{\mathrm{GT}}|^2 |\langle \psi^{(Z)}_{1s}\rangle|^2 \frac{Q^2_{\mathrm{H}}}{\pi}$$

$$E_j(\vec{k}_j) = \frac{M_m^2 - M_d^2 + m_j^2}{2M_m}$$

$$(m_1)_{\mathrm{th}} = 0.22 + 2.29\times 10^{-4}\,\mathrm{eV}$$
$$(m_2)_{\mathrm{th}} = 0.22 + 4.11\times 10^{-4}\,\mathrm{eV}$$
$$(m_3)_{\mathrm{th}} = 0.22 + 5.80\times 10^{-3}\,\mathrm{eV}$$




$$E_i(\vec{k}_i) - E_j(\vec{k}_j) = \frac{m_i^2 - m_j^2}{2M_m}$$

$$(\Delta m^2_{21})_{\mathrm{exp}} = 0.80\times 10^{-4}\,\mathrm{eV}^2$$
$$(\Delta m^2_{32})_{\mathrm{exp}} = 2.40\times 10^{-3}\,\mathrm{eV}^2$$
$$(\Delta m^2_{31})_{\mathrm{exp}} = 2.48\times 10^{-3}\,\mathrm{eV}^2$$

# Time–dependent *EC*–decay rates

**Weak interaction Hamilton operator**

$$\mathrm{H}_W(t) = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3x\, [\bar{\psi}_n(t,\vec{r}\,)\gamma^\mu(1 - g_A\gamma^5)\psi_p(t,\vec{r}\,)]$$

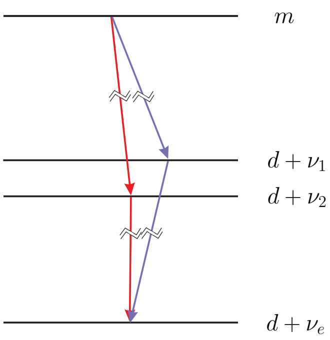
$$\times \sum_j U_{ej}\, [\bar{\psi}_{\nu_j}(t,\vec{r}\,)\gamma_\mu(1 - \gamma^5)\psi_{e^-}(t,\vec{r}\,)]$$

**Amplitudes of *EC*–decays**

$$A(in \to out)(t) = -i \int_{-\infty}^{t} d\tau\, \langle out|H_W(0)|in\rangle\, e^{\,i(E_{out} - E_{in} - i\,\varepsilon)\tau}$$

$$A(m \to d + \nu_e)(t) = \sum_j U_{ej} A(m \to d + \nu_j)(t)$$

# Analogy with quantum beats of atomic transitions (W. W. Chow *et al.*, PRA**11**, 1380 (1975))



# Quantum beat criterion for time–dependence

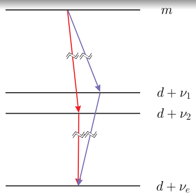
$$\delta E_d \sim \frac{2\pi\hbar}{\tau_d} \gtrsim 4.14 \times 10^{-15}\,\mathrm{eV}$$

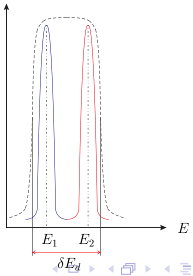
$$\tau_d \lesssim 1\,\mathrm{s}$$

$$\delta E_d \gg E_2(\vec{k}_2) - E_1(\vec{k}_1) =$$

$$= \frac{m_2^2 - m_1^2}{2M_m} \Longrightarrow \frac{2\pi\gamma\hbar}{T_{EC}} = 0.85 \times 10^{-15}\,\mathrm{eV}$$

$$\gamma = 1.43 \quad T_{EC} = 7\,\mathrm{s}$$



# Amplitude of the decay $m \to d + \nu_j$ channel

$$\psi_{\nu_j}(\vec{r},t) = (2\pi\delta^2)^{3/2}\int \frac{d^3k}{(2\pi)^3}\, e^{-\frac{1}{2}\delta^2(\vec{k}-\vec{k}_j)^2}\, e^{\,i\vec{k}\cdot\vec{r} - iE_j(\vec{k})t}\, u_{\nu_j}(\vec{k},\sigma_{\nu_j})$$

$$A(m \to d + \nu_j)(t) = -i\int_{-\infty}^{t} d\tau\, \langle d(\vec{q})\nu_j(\vec{k}_j)|H^{(j)}_W(\tau)|m(\vec{0})\rangle\, e^{\varepsilon\tau} =$$

$$= -\sqrt{3}\,\sqrt{2M_m 2E_d(\vec{q})E_j(\vec{q})}\,\mathcal{M}_{\rm GT}\,\langle\psi^{(Z)}_{1s}\rangle(2\pi\delta^2)^{3/2}$$

$$\times\, e^{-\frac{1}{2}\delta^2(\vec{q}+\vec{k}_j)^2}\, \frac{e^{\,i\,(\Delta E_j(\vec{q}) - i\,\varepsilon)t}}{\Delta E_j(\vec{q}) - i\,\varepsilon}\,\delta_{M_F,-\frac{1}{2}}$$

$$\Delta E_j(\vec{q}) = E_j(\vec{q}) + E_d(\vec{q}) - M_m$$

# Rate of neutrino spectrum of *EC*–decay

$$\frac{dN_{\nu_e}(t)}{dt} =$$

$$= \frac{1}{2M_m}\int \frac{d^3q}{(2\pi)^3 2E_d(\vec{q}\,)}\frac{1}{2F+1}\sum_{M_F=\pm\frac{1}{2}}\frac{d}{dt}|A(m\to d+\nu_e)(t)|^2 =$$

$$= \frac{dN^{(1)}_{\nu_e}(t)}{dt} + \frac{dN^{(2)}_{\nu_e}(t)}{dt}$$

# Rate of neutrino spectrum of *EC*–decay

**$1/\delta$–expansion and rate of neutrino spectrum of *EC*–decay**

$$\frac{dN^{(1)}_{\nu_e}(t)}{dt} \propto \int \frac{d^3q}{(2\pi)^3}\, e^{\,-\delta^2(\vec{q}+\vec{k}_j)^2} F_j(\vec{q}\,) \stackrel{1/\delta}{\Longrightarrow}$$

$$\stackrel{1/\delta}{\Longrightarrow} \frac{1}{(4\pi\delta^2)^{3/2}} F_j(-\vec{k}_j)$$

$$\frac{dN^{(2)}_{\nu_e}(t)}{dt} \propto e^{\,-\frac{1}{4}\,\delta^2(\vec{k}_i-\vec{k}_j)^2} \int \frac{d^3q}{(2\pi)^3}\, e^{\,-\delta^2(\vec{q}+\frac{\vec{k}_i+\vec{k}_j}{2})^2} F_{ij}(\vec{q}\,) \stackrel{1/\delta}{\Longrightarrow}$$

$$\stackrel{1/\delta}{\Longrightarrow} e^{\,-\frac{1}{4}\,\delta^2(\vec{k}_i-\vec{k}_j)^2} \frac{1}{(4\pi\delta^2)^{3/2}} F_{ij}\Big(-\frac{\vec{k}_i+\vec{k}_j}{2}\Big)$$

# Rate of neutrino spectrum of *EC*–decay. Diagonal term

$$\frac{dN^{(1)}_{\nu_e}(t)}{dt} \propto \sum_j |U_{ej}|^2 E_j(\vec{k}_j)\,\frac{2\varepsilon}{(\Delta E_j(\vec{k}_j))^2 + \varepsilon^2} \stackrel{\varepsilon\to 0}{\Longrightarrow}$$

$$\stackrel{\varepsilon\to 0}{\Longrightarrow} \sum_j |U_{ej}|^2 E_j(\vec{k}_j)\,2\pi\,\delta(E_j(\vec{k}_j) + E_d(\vec{k}_j) - M_m)\Big|_{m_j=0} =$$

$$= E_\nu(\vec{k}\,)\,2\pi\,\delta(E_\nu(\vec{k}\,) + E_d(\vec{k}\,) - M_m)$$

**EC–decay rate. Diagonal term**

$$\lambda^{(1)}_{EC} = \frac{1}{2M_m}\int \frac{d^3k}{(2\pi)^3 2E_\nu}\,\frac{1}{(\pi\delta)^{3/2}}\,\frac{dN^{(1)}_{\nu_e}(t)}{dt}\Big|_{m_j=0} = \lambda^{(\mathrm{H})}_{EC}$$

# Rate of neutrino spectrum of *EC*–decay. Interference term

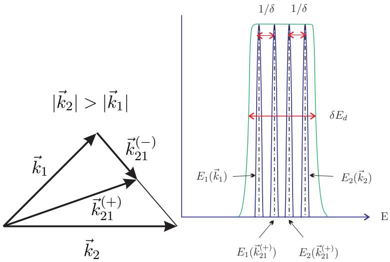
$$\frac{dN^{(2)}_{\nu_e}(t)}{dt} \propto \sum_{i>j} U^*_{ei} U_{ej}\, e^{-\frac{1}{4}\delta^2(\vec{k}_i - \vec{k}_j)^2} \sqrt{E_i(\vec{k}^{(+)}_{ij}) E_j(\vec{k}^{(+)}_{ij})}$$

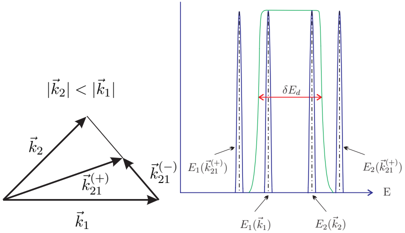
$$\times \left[\frac{2\varepsilon}{(\Delta E_i(\vec{k}_{\{ij\}})^2 + \varepsilon^2} + \frac{2\varepsilon}{(\Delta E_j(\vec{k}^{(+)}_{ij})^2 + \varepsilon^2}\right] \cos\left[\Big(E_i(\vec{k}^{(+)}_{ij}) - E_j(\vec{k}^{(+)}_{ij})\Big) t\right]$$

$$\vec{k}^{(-)}_{ij} = \frac{\vec{k}_i - \vec{k}_j}{2} \qquad \vec{k}^{(+)}_{ij} = \frac{\vec{k}_i + \vec{k}_j}{2}$$

# Frequencies of interference term of *EC*–decay

$$E_1 < E_1(\vec{k}_{21}) < E_2(\vec{k}_{21}) < E_2 \qquad E_1(\vec{k}^{(+)}_{21}) < E_1 < E_2 < E_2(\vec{k}^{(+)}_{21})$$




$$E_2(\vec{k}_2) - E_1(\vec{k}_1) = \omega_{21} = \frac{\Delta m^2_{21}}{2M_m} \qquad E_2(\vec{k}_2) - E_1(\vec{k}_1) = \Omega_{21} = \frac{\Delta m^2_{21}}{2Q_{\mathrm{H}}}$$

# Probability density $\rho(\vec{k}_2, \vec{k}_1)$

**Probability density**

$$\rho(\vec{k}_2, \vec{k}_1) = \theta(|\vec{k}_2| - |\vec{k}_1|)\, \rho'(\vec{k}_2, \vec{k}_1) + \theta(|\vec{k}_1| - |\vec{k}_2|)\, \rho''(\vec{k}_2, \vec{k}_1),$$

**Massless limit of probability density**

$$\lim_{m_1, m_2 \to 0} \rho(\vec{k}_2, \vec{k}_1) = 2\sqrt{E_\nu(\vec{k}_1) E_\nu(\vec{k}_2)}(2\pi)^3 \delta^{(3)}(\vec{k}_2 - \vec{k}_1)$$

# Definition of interference term

**Definition of interference term**

$$\lambda^{(2)}_{EC}(t) = \int \frac{1}{(\pi\delta^2)^{3/2}}\,\frac{dN^{(2)}_{\nu_e}(t)}{dt}\,\rho(\vec{k}_2,\vec{k}_1)\,\frac{d^3k_1}{(2\pi)^3 2E_1(\vec{k}_1)}\,\frac{d^3k_2}{(2\pi)^3 2E_2(\vec{k}_2)}$$

**Result of calculation of interference term**

$$\lambda^{(2)}_{EC}(t) = \lambda^{(2)'}_{EC}(t) + \lambda^{(2)''}_{EC}(t)$$

# Low–frequency contribution to interference term

$$\lambda^{(2)\prime}_{EC}(t) \propto \int \frac{d^3k_1}{(2\pi)^3 2E_1(\vec{k}_1)} \frac{d^3k_2}{(2\pi)^3 2E_2(\vec{k}_2)}$$

$$\times\, \theta(|\vec{k}_2| - |\vec{k}_1|)\, \rho'(\vec{k}_2, \vec{k}_1)\, e^{-\frac{1}{2}\delta^2(\vec{k}_2 - \vec{k}_1)^2} \sqrt{E_2(\vec{k}^{(+)}_{21}) E_1(\vec{k}^{(+)}_{21})}$$

$$\times \left[\frac{2\varepsilon}{(\Delta E_2(\vec{k}^{(+)}_{21}))^2 + \varepsilon^2} + \frac{2\varepsilon}{(\Delta E_1(\vec{k}^{(+)}_{21}))^2 + \varepsilon^2}\right]$$

$$\times \cos\left[\Big(E_2(\vec{k}^{(+)}_{21}) - E_1(\vec{k}^{(+)}_{21})\Big) t\right]$$

# High–frequency contribution to interference term

$$\lambda^{(2)''}_{EC}(t) \propto \int \frac{d^3k_1}{(2\pi)^3 2E_1(\vec{k}_1)} \frac{d^3k_2}{(2\pi)^3 2E_2(\vec{k}_2)}$$

$$\times\, \theta(|\vec{k}_1| - |\vec{k}_2|)\, \rho''(\vec{k}_2, \vec{k}_1)\, e^{-\frac{1}{4}\delta^2(\vec{k}_2 - \vec{k}_1)^2} \sqrt{E_2(\vec{k}^{(+)}_{21}) E_1(\vec{k}^{(+)}_{21})}$$

$$\times \Big[\frac{2\varepsilon}{(\Delta E_2(\vec{k}^{(+)}_{21}))^2 + \varepsilon^2} + \frac{2\varepsilon}{(\Delta E_1(\vec{k}^{(+)}_{21}))^2 + \varepsilon^2}\Big]$$

$$\times \cos\Big[\Big(\frac{m_2^2 - m_1^2}{E_2(\vec{k}^{(+)}_{21}) + E_1(\vec{k}^{(+)}_{21})}\Big)t\Big]$$

# $1/\delta$ expansions of contributions to interference term

$$E_2(\vec{k}_2) \simeq E_1(\vec{k}_1) \gg |\vec{k}^{(-)}_{21}| = \left|\frac{\vec{k}_2 - \vec{k}_1}{2}\right| \sim \frac{1}{\delta}$$

$$E_2(\vec{k}^{(+)}_{21}) \to E_2(\vec{k}_2) - \frac{\vec{k}^{(-)}_{21}\cdot\vec{k}_2}{E_2(\vec{k}_2)} = E_2(\vec{k}_2) - O\Big(\frac{1}{\delta}\Big)$$

$$E_1(\vec{k}^{(+)}_{21}) \to E_1(\vec{k}_1) + \frac{\vec{k}^{(-)}_{21}\cdot\vec{k}_1}{E_1(\vec{k}_1)} = E_1(\vec{k}_1) + O\Big(\frac{1}{\delta}\Big)$$

$$\Delta E_2(\vec{k}^{(+)}_{21}) = E_2(\vec{k}^{(+)}_{21}) + E_d(\vec{k}^{(+)}_{21}) - M_m \to \Delta E_2(\vec{k}_2) - O\Big(\frac{1}{\delta}\Big)$$

$$\Delta E_1(\vec{k}^{(+)}_{21}) = E_1(\vec{k}^{(+)}_{21}) + E_d(\vec{k}^{(+)}_{21}) - M_m \to \Delta E_1(\vec{k}_1) + O\Big(\frac{1}{\delta}\Big)$$

# Low–frequency contribution to interference term

$$\lambda^{(2)'}_{EC}(t) \propto \int \frac{d^3k_1}{(2\pi)^3 2E_1(\vec{k}_1)}\,\frac{d^3k_2}{(2\pi)^3 2E_2(\vec{k}_2)}$$

$$\times\,\theta(|\vec{k}_2| - |\vec{k}_1|)\,\rho'(\vec{k}_2,\vec{k}_1)\sqrt{E_2(\vec{k}_2)E_1(\vec{k}_1)}$$

$$\times\, e^{\,-\frac{1}{4}\delta^2(\vec{k}_2 - \vec{k}_1)^2}$$

$$\times\Big[2\pi\,\delta(E_2(\vec{k}_2) + E_d(\vec{k}_2) - M_m) + 2\pi\,\delta(E_1(\vec{k}_1) + E_d(\vec{k}_1) - M_m)\Big]$$

$$\times\,\cos\Big[\Big(E_2(\vec{k}_2) - E_1(\vec{k}_1)\Big)t\Big]$$

$$\omega_{21} = E_2(\vec{k}_2) - E_1(\vec{k}_1) = \frac{\Delta m^2_{21}}{2M_m}$$

# High–frequency contribution to interference term

$$\lambda^{(2)''}_{EC}(t) \propto \int \frac{d^3k_1}{(2\pi)^3 2E_1(\vec{k}_1)}\frac{d^3k_2}{(2\pi)^3 2E_2(\vec{k}_2)}$$

$$\times\,\theta(|\vec{k}_1| - |\vec{k}_2|)\,\rho''(\vec{k}_2,\vec{k}_1)\,\sqrt{E_2(\vec{k}_2)E_1(\vec{k}_1)}$$

$$\times\, e^{\,-\frac{1}{4}\delta^2(\vec{k}_2 - \vec{k}_1)^2}$$

$$\times\Big[2\pi\,\delta(E_2(\vec{k}_2) + E_d(\vec{k}_2) - M_m) + 2\pi\,\delta(E_1(\vec{k}_1) + E_d(\vec{k}_1) - M_m)\Big]$$

$$\times\,\cos\Big[\Big(\frac{m_2^2 - m_1^2}{E_2(\vec{k}_2) + E_1(\vec{k}_1)}\Big)t\Big]$$

$$\Omega_{21} = \frac{m_2^2 - m_1^2}{E_2(\vec{k}_2) + E_1(\vec{k}_1)} \simeq \frac{\Delta m^2_{21}}{2Q_{\rm H}} \qquad E_2(\vec{k}_2) \simeq E_1(\vec{k}_1) \simeq Q_{\rm H}$$

# Time–dependent *EC*–decay rate

**$EC$–decay rate in the rest frame of mother ion**

$$\frac{\lambda_{EC}(t)}{\lambda_{EC}} = 1 + a_{EC}\,\cos(\omega_{21}t) + \tilde{a}_{EC}\,\cos(\Omega_{21}t)$$

**$EC$–decay rate in the laboratory frame**

$$\frac{\lambda_{EC}(t)}{\lambda_{EC}} = 1 + a_{EC}\,\cos\left(\frac{2\pi t}{T_{EC}}\right) + \tilde{a}_{EC}\,\cos\left(\frac{2\pi t}{T_G}\right)$$

**Periods of time-dependence**

$$T_{EC} = \frac{2\pi\gamma}{\omega_{21}} = \frac{4\pi\gamma M_m\hbar}{\Delta m^2_{21}} \qquad T_G = \frac{2\pi\gamma}{\Omega_{21}} = \frac{4\pi\gamma Q_{\rm H}\hbar}{\Delta m^2_{21}}$$

# Amplitudes of time–dependent *EC*–decay rate

## Amplitudes of periodic time–dependence

$$\begin{pmatrix} a_{EC} \\ \tilde{a}_{EC} \end{pmatrix} = \sin 2\theta_{12}\, \frac{2\pi^2}{Q^2_{\rm H}} \int \frac{d^3k_1}{(2\pi)^3 2E_1(\vec{k}_1)} \frac{d^3k_2}{(2\pi)^3 2E_2(\vec{k}_2)}$$

$$\times \begin{pmatrix} \theta(|\vec{k}_2| - |\vec{k}_1|)\, \rho'(\vec{k}_2, \vec{k}_1) \\ \theta(|\vec{k}_1| - |\vec{k}_2|)\, \rho''(\vec{k}_2, \vec{k}_1) \end{pmatrix} e^{-\frac{1}{4}\,\delta^2(\vec{k}_2 - \vec{k}_1)^2} \sqrt{E_2(\vec{k}_2) E_1(\vec{k}_1)}$$

$$\times \Big[2\pi\,\delta(E_2(\vec{k}_2) + E_d(\vec{k}_2) - M_m) + 2\pi\,\delta(E_1(\vec{k}_1) + E_d(\vec{k}_1) - M_m)\Big] =$$

$$= \begin{pmatrix} p\,\sin 2\theta_{12} \\ (1-p)\,\sin 2\theta_{12} \end{pmatrix}$$

# Influence of off–shell properties of massive neutrinos

- Due to non–conservation of 3–momenta massive neutrinos are off–shell. This means that on–shell relations $E_j(\vec{p}\,) = (\vec{p}^{\,2} + m_j^2)^{1/2}$ between energies $E_j(\vec{p}\,)$ and 3-momenta $\vec{p} = \vec{k}_j$ or $\vec{p} = \vec{k}_{ij}^{(+)}$ are only approximate.
- For the correct calculation of the periodic dependence on time one cannot use on–shell relations between energies and momenta of massive neutrinos for the analysis of the differences of neutrino energies and momenta due to their sensitivity to off–shell and on–shell states. For example, see A. Gal: 0809.1213 [nucl–th].

# Experimental time–spectrum and KamLAND problem

## Experimental time–spectrum of $EC$–decay rate

$$\Delta T = 5 \times 0.064\,\mathrm{s} = 0.32\,\mathrm{s} \Longrightarrow N_G = \frac{\Delta T}{T_G} \sim 1800$$

$$\lambda^{(\mathrm{H})}_{EC}(t) = \lambda^{(\mathrm{H})}_{EC}\Big\{1 + a_{EC}\cos\Big(\frac{2\pi}{T_{EC}}t\Big)\Big\}$$

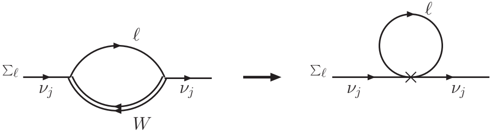
## KamLAND problem

$$T_{EC} = \frac{4\pi\gamma\hbar M_m}{\Delta m^2_{21}} = 7\,\mathrm{s} \Longrightarrow (\Delta m^2_{21})_{\mathrm{GSI}} = 2.22(3)\times 10^{-4}\,\mathrm{eV}^2$$

$$(\Delta m^2_{21})_{\mathrm{GSI}} = 2.22(3)\times 10^{-4}\,\mathrm{eV}^2 > (\Delta m^2_{21})_{\mathrm{KamLAND}} = 0.80(5)\times 10^{-4}\,\mathrm{eV}^2$$

# Solution of KamLAND problem

**Assumption: Neutrinos get mass–corrections** $E_{\nu_j}(r) = \sqrt{\vec{k}_j^{\,2} + (m_j + \delta m_j(r))^2}$, **caused by a strong Coulomb field of the daughter ion:** $\nu_j \to \sum_\ell U^*_{j\ell}\ell^- W^+$

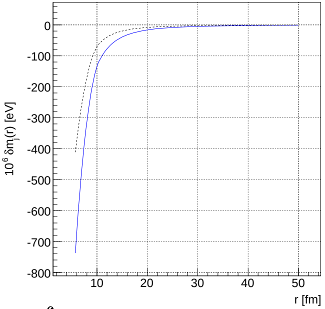



$$A(m \to d + \nu_j)(t) \propto \int d^3x\, \Psi^*_d(r)\Psi_m(r)\psi^{(Z)}_{1s}(r)\, e^{i(E_d + E_{\nu_j}(r) - M_m)t}$$

$$\propto e^{i(E_d + E_{\nu_j}(R) - M_m)t}\,\langle\psi^{(Z)}_{1s}\rangle\,\mathcal{M}_{\rm GT}$$

# Solution of KamLAND problem

## Neutrino mass–corrections in strong Coulomb field




$$A(m \to d + \nu_j)(t) \propto \int d^3x\, \Psi^*_d(r)\Psi_m(r)\psi^{(Z)}_{1s}(r)\, e^{i(E_d + E_{\nu_j}(r) - M_m)t}$$

$$\propto e^{i(E_d + E_{\nu_j}(R) - M_m)t}\langle\psi^{(Z)}_{1s}\rangle\,\mathcal{M}_{\rm GT} \quad R = 5.712\,\mathrm{fm}$$

# Neutrino masses from the “Darmstadt oscillations”

$$\delta m_1(R) = -7.37\times 10^{-4}\,\mathrm{eV} \quad \delta m_2(R) = -4.11\times 10^{-4}\,\mathrm{eV}$$

$$T_{EC} = \frac{4\pi\gamma M_m}{(m_2+\delta m_2(R))^2-(m_1+\delta m_1(R))^2}$$

$$2m_2\delta m_2(R) - 2m_1\delta m_1(R) = (\Delta m^2_{21})_{\mathrm{GSI}} - (\Delta m^2_{21})_{\mathrm{KamLAND}}$$

$$(m_1)_{\mathrm{th}} = 0.22 + 2.29\times 10^{-4}\,\mathrm{eV}$$
$$(m_2)_{\mathrm{th}} = 0.22 + 4.11\times 10^{-4}\,\mathrm{eV}$$
$$(m_3)_{\mathrm{th}} = 0.22 + 5.80\times 10^{-3}\,\mathrm{eV} \to \sum_{j=1,2,3}(m_j)_{\mathrm{th}} \simeq 0.67\,\mathrm{eV} < 1\,\mathrm{eV}$$

$$(\Delta m^2_{21})_{\mathrm{exp}} = 0.80\times 10^{-4}\,\mathrm{eV}$$
$$(\Delta m^2_{32})_{\mathrm{exp}} = 2.40\times 10^{-3}\,\mathrm{eV}$$

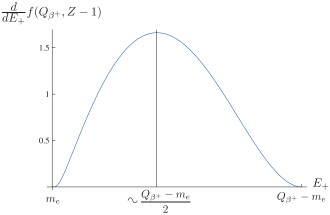
# Nuclear positron ($\beta^+$) decays of H–like heavy ions

$${}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{57+} + e^+ + \nu_e \qquad {}^{142}\mathrm{Pm}^{60+} \to {}^{142}\mathrm{Nd}^{59+} + e^+ + \nu_e$$

**Neutrino and positron energy spectra are continuous**

$$E_\nu + E_+ = Q_{\beta^+} - m_e$$

$$\frac{d}{dE_+} f(Q_{\beta^+}, Z-1) = E_+ \sqrt{E_+^2 - m_e^2}\,(Q_{\beta^+} - m_e - E_+)^2 F(Z-1, E_+)$$



# Time–dependence of $\beta^+$–decay rates of H–like heavy ions (PRL **101**, 182501 (2008))

**Amplitudes of the $\beta^+$–decay of the H–like heavy ions**

$$A(m \to d + e^+ + \nu_e)(t)_{FM_F \to F'M_{F'}}(t) =$$

$$= \sum_j U_{ej} A(m \to d + e^+ + \nu_j)(t)_{FM_F \to F'M_{F'}}(t)$$

$$A(m \to d + e^+ + \nu_j)(t)_{FM_F \to F'M_{F'}}(t) =$$

$$= -i \int_{-\infty}^{t} d\tau \, \langle (d(\vec{q}\,))_{F'M_{F'}} e^+(\vec{p}_+) \nu_j(\vec{k}_j) | H^{(j)}_W(\tau) | (m(\vec{0}\,))_{FM_F} \rangle$$

# Time–dependence of $\beta^+$–decay rates of H–like heavy ions: $(m)_{FM_F} \to (d)_{F'M_{F'}} + e^+ + \nu_e$

**Rate of neutrino energy spectrum of the $\beta^+$–decay of the H–like heavy ions**

$$\frac{dN_{\nu_e}(t)}{dt} = \frac{1}{2M_m}\frac{1}{2F+1}\frac{d}{dt}\int\sum_{M_F,M_{F'}}|A(m \to d+e^+ +\nu_e)(t)_{FM_F \to F'M_{F'}}|^2$$

$$\times F(Z-1,E_+)\frac{d^3k_d}{(2\pi)^3 2E_d}\frac{d^3p_+}{(2\pi)^3 2E_+}$$

# Time–dependence of $\beta^+$–decay rates of H–like heavy ions: $(m)_{FM_F} \to (d)_{F'M_{F'}} + e^+ + \nu_e$

**Rate of neutrino energy spectrum of the $\beta^+$–decay**

$$\frac{dN_{\nu_e}(t)}{dt} = \frac{1}{2F+1}\,\frac{|\mathcal{M}_{\rm GT}|^2}{\pi^2}\,(\pi\delta^2)^{3/2}$$

$$\times \int_{m_e}^{Q_{\beta^+}-m_e} dE_+\,E_+\,\sqrt{E_+^2 - m_e^2}\,F(Z-1, E_+)$$

$$\times\,(2\pi)\,\delta(Q_{\beta^+} - m_e - E_+ - E_\nu)\,E_\nu$$

$$\times\Big\{1 + \sum_{i>j} 2U^*_{ei}U_{ej}\,\cos\Big[\Big(\sqrt{E^2_\nu + m^2_i} - \sqrt{E^2_\nu + m^2_j}\Big)t\Big]\Big\}$$

# Time–dependence of $\beta^+$–decay rates of H–like heavy ions: $(m)_{FM_F} \to (d)_{F'M_{F'}} + e^+ + \nu_e$

**The $\beta^+$–decay rate of the H–like heavy ions**

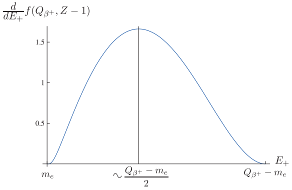
$$\lambda^{(\rm H)}_{\beta^+}(t) = \int \frac{d^3k}{(2\pi)^3 2E_\nu}\,\frac{1}{(\pi\delta^2)^{3/2}}\,\frac{dN_\nu(t)}{dt} = \lambda^{(\rm H)}_{\beta^+}(1 + R_{\beta^+}(t))$$

**Time-dependent part of the $\beta^+$–decay rate for $\theta_{13} = 0$**

$$R_{\beta^+}(t) \propto \int_{m_e}^{Q_{\beta^+}-m_e} dE_+ E_+\sqrt{E^2_+ - m^2_e}\,(Q_{\beta^+} - m_e - E_+)^2\,\frac{F(Z-1, E_+)}{f(Q_{\beta^+}, Z-1)}$$
$$\times\,\cos\Big[\Big(\sqrt{(Q_{\beta^+} - m_e - E_+)^2 + m^2_2} - \sqrt{(Q_{\beta^+} - m_e - E_+)^2 + m^2_1}\Big)t\Big]$$

# Time–dependence of $\beta^+$–decay rates of H–like heavy ions: $(m)_{FM_F} \to (d)_{F'M_{F'}} + e^+ + \nu_e$

$$\frac{d}{dE_+} f(Q_{\beta^+}, Z-1) = E_+ \sqrt{E_+^2 - m_e^2}\,(Q_{\beta^+} - m_e - E_+)^2 F(Z-1, E_+)$$

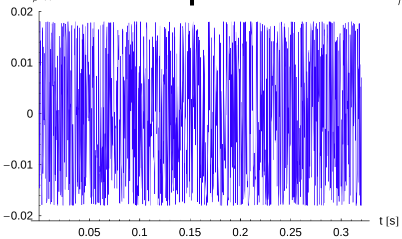



**Period of the time–dependence of the $\beta^+$–decay rate**

$$T_{\beta^+} \sim \frac{2\pi\gamma\hbar(Q_{\beta^+} - m_e)}{(\Delta m^2_{21})_{\rm GSI}} \sim 8 \times 10^{-5}\,{\rm s}$$

# Time–dependence of $\beta^+$–decay rates of H–like heavy ions

**On the experimental time–spectrum of the $\beta^+$–decay rate**




$$N_{5\times bin} = \frac{\Delta T}{T_{\beta+}} = \frac{0.32}{T_{\beta+}} \sim 4000 \Longrightarrow \lambda^{(\mathrm{H})}_{\beta+}(t)_{\mathrm{exp}} = \lambda^{(\mathrm{H})}_{\beta+}$$

**Bosch, report at EXA08, Kienle, report at PANIC08**

# Summary. Conditions for existence of interference term in *EC*–decay rates

- Coherence of massive neutrinos $\nu_j$ in the final state of the *EC*–decay, caused by the entanglement of electron only with the electron neutrino $\nu_e$, which is a coherent superposition $|\nu_e\rangle = \sum_j U^*_{ej}|\nu_j\rangle$ of massive neutrinos $\nu_j$,
- Non–conservation of 3–momenta of massive neutrino leading to the off–shell states of massive neutrinos,
- Energy conservation, imposed by the *Fermi Golden Rule*,
- Overlap of energy levels of massive neutrinos due to the energy uncertainty $\delta E_d \sim 2\pi\hbar/\tau_d$, caused by the experimental time differential detection of the daughter ions during an interim $\tau_d$

# Summary. Conditions for non–existence of interference term in *EC*–decay rates: 0807.2750 [nucl-th]

- Conservation of energy and 3–momenta for all *EC*–decay channels $m \to d + \nu_j$ with neutrino mass–eigenstates $\nu_j$

**Cohen, Glashow, Ligeti,** *Disentangling Neutrino Oscillations*, **arXiv: 0810.4602 [hep-ph]**

# On the approaches by C. Giunti (PLB **665**,92(2008) and H. Kienert *et al.*, arXiv: 0808.2389 [hep–ph]

**Starting assumption**

Analogy with quantum beats of atomic transitions: Two closely spaced mass-eigenstates $|m'\rangle$ and $|m''\rangle$ of the mother ion in the coherent state

$$|m\rangle = \cos\theta\,|m'\rangle + \sin\theta\,|m''\rangle$$

decay into one final state $|d\,\nu_e\rangle$

# On the approaches by C. Giunti (PLB **665**,92(2008) and H. Kienert *et al.*, arXiv: 0808.2389 [hep–ph]

**Expected consequence: 0811.0922 [nucl-th]**

$$\lambda^{(m)}_{EC}(t) = \lambda_{EC}(1 + 2\sin 2\theta \cos(\Delta E_{m'm''} t)) \quad T_{EC} \sim 1/\Delta E_{m'm''}$$

# On the approaches by C. Giunti (PLB **665**,92(2008) and H. Kienert *et al.*, arXiv: 0808.2389 [hep–ph]

**Unexpected consequence 1: 811.0922 [nucl-th]**

$$\lambda^{(m)}_{\beta^+}(t) = \lambda_{\beta^+}(1+2\sin 2\theta\,\cos(\Delta E_{m'm''}t)) \quad T_{\beta^+} = T_{EC} \sim 1/\Delta E_{m'm''}$$

- In contradiction with the experimental data at GSI

**Bosch, report at EXA08, Kienle, report at PANIC08**

# On the approaches by C. Giunti (PLB **665**, 92 (2008) and H. Kienert *et al.*, arXiv: 0808.2389 [hep–ph]

**Unexpected consequence 2: 0811.0922 [nucl-th]**

Statistical population of the mass–eigenstates $|m'\rangle$ and $|m''\rangle$ leading to the production of the coherent state

$$|\tilde{m}\rangle = -\sin\theta\,|m'\rangle + \cos\theta\,|m''\rangle$$

with the decay rates:

$$\lambda^{(\tilde{m})}_{EC}(t) = \lambda_{EC}(1 - 2\sin 2\theta\cos(\Delta E_{m'm''}t))$$

$$\lambda^{(\tilde{m})}_{\beta^+}(t) = \lambda_{\beta^+}(1 - 2\sin 2\theta\,\cos(\Delta E_{m'm''}t))$$

# On the approaches by C. Giunti (PLB **665**, 92 (2008) and H. Kienert *et al.*, arXiv: 0808.2389 [hep–ph]

$$\lambda_{EC}(t) = P_m \lambda^{(m)}_{EC}(t) + P_{\tilde m}\, \lambda^{(\tilde m)}_{EC}(t) =$$

$$= \lambda_{EC}(1{+}2\sin 2\theta\,(P_m{-}P_{\tilde m})\cos(\Delta E_{m'm''}t)) = \lambda_{EC} \quad P_m = P_{\tilde m} = \frac{1}{2}$$

$$\lambda_{\beta^+}(t) = P_m \lambda^{(m)}_{\beta^+}(t) + P_{\tilde m}\, \lambda^{(\tilde m)}_{\beta^+}(t) =$$

$$= \lambda_{\beta^+}(1{+}2\sin 2\theta\,(P_m{-}P_{\tilde m})\cos(\Delta E_{m'm''}t)) = \lambda_{\beta^+} \quad P_m = P_{\tilde m} = \frac{1}{2}$$

**Unexpected consequence 3: 0811.0922 [nucl-th]**

- No time–dependence of both *EC* and $\beta^+$ decay rates