Mössbauer neutrinos

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Workshop on neutrinos in particle, in nuclear and in astrophysics, Trento,
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MAX-PLANCK-GESELLSCHAFT

in collaboration with E. Kh. Akhmedov and M. Lindner
based on JHEP **0805** (2008) 005 (arXiv:0802.2513)
and arXiv:0803.1424

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Outline

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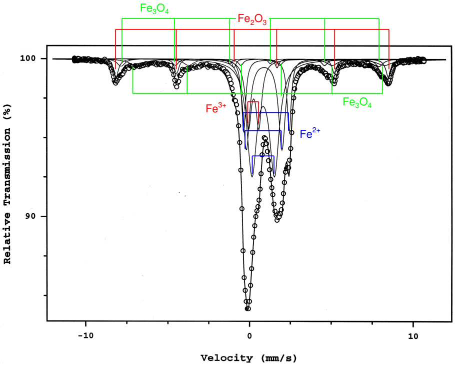

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A similar effect should exist for neutrino emission/absorption in bound state β decay and induced electron capture processes.

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Proposed experiment:

Production: $^3\text{H} \rightarrow {}^3\text{He}^+ + \bar{\nu}_e + e^-(\text{bound})$

Detection: $^3\text{He}^+ + e^-(\text{bound}) + \bar{\nu}_e \rightarrow {}^3\text{H}$

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Physics goals:

- Neutrino oscillations on a laboratory scale: $E = 18.6$ keV, $L^{\mathrm{osc}}_{\mathrm{atm}} \sim 20$ m.
- Gravitational interactions of neutrinos
- Study of solid state effects with unprecedented precision

Mössbauer neutrinos (2)

Mössbauer neutrinos have very special properties:

- Neutrino receives *full* decay energy: $Q = 18.6$ keV
- Natural line width: $\gamma \sim 1.17 \times 10^{-24}$ eV
- Atucal line width: $\gamma \gtrsim 10^{-11}$ eV
  - Inhomogeneous broadening (Impurities, lattice defects)
  - Homogeneous broadening (Spin interactions)

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Experimental challenges:

- Is the Lamb-Mössbauer factor (fraction of recoil-free emissions/absorptions) large enough?
- Can a linewidth $\gamma \gtrsim 10^{-11}$ eV be achieved?
- Can the resonance condition be fulfilled?

Mössbauer neutrinos (3)

Recent controversy:

- Does the small energy uncertainty prohibit oscillations of Mössbauer neutrinos?
- Do oscillating neutrinos need to have equal energies resp. equal momenta?

S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G34** (2007) 987, hep-ph/0611285

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⇒ Careful treatment with as few assumptions as possible is needed
⇒ Answer to the above questions will be No.

Outline

Equal energies or equal momenta?

Typical *assumptions* in the "textbook derivation" of the oscillation formula:

- Different mass eigenstates have equal energies: $E_j = E_k \equiv E$

  ("Evolution only in space", "Stationary evolution")

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These are *assumptions* or *approximations*, not fundamental principles!

Equal energies or equal momenta? (2)

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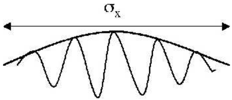
Beuthe, Giunti, Grimus, Kiers, Kim, Lee, Mohanty, Nussinov, Stockinger, Weiss, ...

Conditions for oscillations in a wave packet approach



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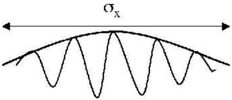

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  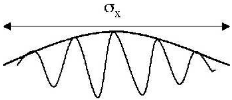

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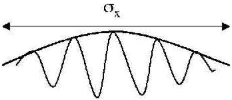

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B. Kayser, Phys. Rev. **D24** (1981) 110

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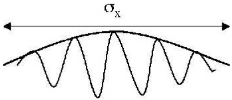

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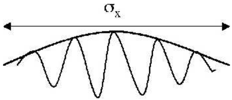
This is easily fulfilled for Mössbauer neutrinos, since

$$\sigma_E \sim 10^{-11}\ \text{eV}$$
$$\sigma_p = 1/2\sigma_x \sim 1/\text{interatomic distance} \sim 10\ \text{keV}$$
$$E = p = 18.6\ \text{keV}$$

Conditions for oscillations in a wave packet approach

- Coherence in production and detection processes
- Coherence maintained during propagation



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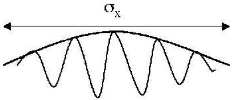

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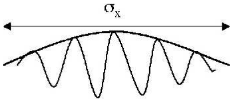

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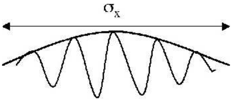

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$\Rightarrow$ Stanard oscillation formula is approximately recovered:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\Big[-2\pi i \frac{L}{L^{\mathrm{osc}}_{jk}} \Big]$$

$$L^{\mathrm{osc}}_{jk} = \frac{4\pi E}{\Delta m^2_{jk}}$$

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Aim: Properties of the neutrino should be automatically determined from properties of the source and the detector.

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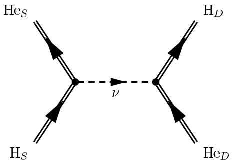

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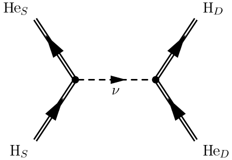

External particles reside in harmonic oscillator potentials.
E.g. for ^{3}H atoms in the source:

$$\psi_{\mathrm{H},S}(\vec{x},t) = \left[\frac{m_\mathrm{H}\omega_{\mathrm{H},S}}{\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_\mathrm{H}\omega_{\mathrm{H},S}|\vec{x}-\vec{x}_S|^2\right]\cdot e^{-iE_{\mathrm{H},S}t}$$

Oscillation amplitude

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2 \left(\frac{m_{\mathrm{H}}\omega_{\mathrm{H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\mathrm{H}}\omega_{\mathrm{H},S}|\vec{x}_1-\vec{x}_S|^2\right] e^{-iE_{\mathrm{H},S}t_1} \\
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\cdot \sum_j \mathcal{M}^{\mu}\mathcal{M}^{\nu *}|U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)} \\
\cdot \bar{u}_{e,S}\gamma_\mu(1-\gamma^5)\,\frac{i(\not{p}+m_j)}{p_0^2-\vec{p}^2-m_j^2+i\epsilon}\,(1+\gamma^5)\gamma_\nu u_{e,D}.
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Evaluation:

- $dt_1\, dt_2$-integrals $\rightarrow$ energy-conserving δ functions $\rightarrow$ p_0-integral trivial
- $d^3x_1\, d^3x_2$-integrals are Gaussian
- d^3p-integral: Use Grimus-Stockinger theorem (for large $L = |\vec{x}_D - \vec{x}_S|$).

W. Grimus, P. Stockinger, Phys. Rev. **D54** (1996) 3414, hep-ph/9603430

From the amplitude to the transition rate

Amplitude:

$$i\mathcal{A} = \frac{-i}{2L}\mathcal{N}\,\delta(E_S - E_D)\,\exp\left[-\frac{E_S^2 - m_j^2}{2\sigma_p^2}\right]\sum_j \mathcal{M}^\mu \mathcal{M}^{\nu *}|U_{ej}|^2\, e^{i\sqrt{E_S^2-m_j^2}L}$$

$$\cdot\,\bar{u}_{e,S}\gamma_\mu \tfrac{1-\gamma^5}{2}(\not{p}_j + m_j)\tfrac{1+\gamma^5}{2}\gamma_\nu u_{e,D},$$

$$\sigma_p^{-2} = (m_{\mathrm{H}}\omega_{\mathrm{H},S} + m_{\mathrm{He}}\omega_{\mathrm{He},S})^{-1} + (m_{\mathrm{H}}\omega_{\mathrm{H},D} + m_{\mathrm{He}}\omega_{\mathrm{He},D})^{-1}$$

From the amplitude to the transition rate

Amplitude:

$$i\mathcal{A} = \frac{-i}{2L}\mathcal{N}\,\delta(E_S - E_D)\,\exp\left[-\frac{E_S^2 - m_j^2}{2\sigma_p^2}\right]\sum_j \mathcal{M}^{\mu}\mathcal{M}^{\nu *}|U_{ej}|^2\, e^{i\sqrt{E_S^2 - m_j^2}L}$$

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Transition rate: Integrate $|\mathcal{A}|^2$ over densities of initial and final states

$$\Gamma \propto \int_0^{\infty} dE_{{\rm H},S}\, dE_{{\rm He},S}\, dE_{{\rm He},D}\, dE_{{\rm H},D}$$

$$\cdot\,\delta(E_S - E_D)\rho_{{\rm H},S}(E_{{\rm H},S})\,\rho_{{\rm He},D}(E_{{\rm He},D})\,\rho_{{\rm He},S}(E_{{\rm He},S})\,\rho_{{\rm H},D}(E_{{\rm H},D})$$

$$\cdot\sum_{j,k}|U_{ej}|^2|U_{ek}|^2\,\underbrace{\exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right]}_{\text{Analogue of Lamb-Mössbauer factor (Recoil-free fraction)}}\,\underbrace{e^{i\left(\sqrt{E_S^2 - m_j^2} - \sqrt{E_S^2 - m_k^2}\right)L}}_{\text{Oscillation phase}}$$

The Lamb-Mössbauer factor

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Convenient reformulation:

$$\exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right] = \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right]\exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

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$\Rightarrow$ Localization condition

$$4\pi\sigma_x E/\sigma_p \lesssim L_{jk}^{\text{osc}},$$

(with $\sigma_x = 1/2\sigma_p$) is satisfied if $L_{jk}^{\text{osc}} \gtrsim 2\pi\sigma_x$, which is easily fulfilled in realistics situations.

Line broadening

Energy levels of ^{3}H and ^{3}He in the source and detector are smeared e.g. due to spin-spin interactions, crystal impurities, lattice defects, etc.

R. S. Raghavan, hep-ph/0601079

W. Potzel, Phys. Scripta **T127** (2006) 85

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$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$

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Result for two neutrino flavours:

$$\Gamma \propto \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S+\gamma_D)^2}{4}} \cdot \left\{ 1 - 2s^2c^2 \left[1 - \frac{1}{2}(e^{-L/L_S^{\mathrm{coh}}} + e^{-L/L_D^{\mathrm{coh}}}) \cos\left(\pi \frac{L}{L^{\mathrm{osc}}}\right)\right]\right\}$$

$$L_{S,D}^{\mathrm{coh}} = 4\bar{E}^2/\Delta m^2 \gamma_{S,D}$$

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$$L_{S,D}^{\rm coh} = 4\bar{E}^2/\Delta m^2 \gamma_{S,D}$$

In realistic cases: $L_{S,D}^{\rm coh} \gg L^{\rm osc} \Rightarrow$ Decoherence is not an issue.

Outline

The time-energy uncertainty relation

Mandelstam-Tamm relation:

$$\Delta E \Delta O \geq \frac{1}{2} \left| \frac{d}{dt} \overline{O}(t) \right| .$$

Here, $\overline{O}(t) = \langle \psi(t) | O | \psi(t) \rangle$, for any operator O and QFT Fock state $|\psi(t)\rangle$.

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Choose $O \equiv |\nu_\alpha\rangle\langle\nu_\alpha|$ (projection in 3d flavour space) and $\psi(t)$ a neutrino state

$$\Delta E \geq \frac{1}{2}\frac{|\frac{d}{dt}P(t)|}{\sqrt{P(t) - P^2(t)}},$$

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S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G35** (2008) 095003 (arXiv:0803.0527)

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Problem: Interpretation of $P(t)$.

- Would be the oscillation probability for a completely delocalized detector.
- Imagine wave packet which is large compared to L^{osc}: Delocalized detector averages out oscillations.
- More realistic: Projection in flavour space and in coordinate space:

$$O_{\vec{x}} \equiv |\nu_\alpha\rangle|\vec{x}\rangle\langle\vec{x}|\langle\nu_\alpha|$$

The time-energy uncertainty relation (2)

Mandelstam-Tamm relation now reads:

$$\Delta E \geq \frac{1}{2} \frac{|\frac{d}{dt} P(\vec{x}, t)|}{\sqrt{P(\vec{x}, t) - P^2(\vec{x}, t)}} ,$$

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Two-flavour approximation: $P(\vec{x}, t) = 1 - \sin^2 2\theta \, \sin^2 \phi(\vec{x}, t)$

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"Energy difference of mass eigenstates must be smaller than energy uncertainty."

Easily fulfilled for Mössbauer neutrinos due to large *momentum* uncertainty.

Outline

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  - Both conditions are easily fulfilled in realistic experiments.
- The time energy uncertainty relation does not inhibit oscillations of Mössbauer neutrinos.

Thank you!

Textbook derivation of the oscillation formula

Diagonalization of the mass terms of the charged leptons and neutrinos gives

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} \left(\bar{e}_{\alpha L} \gamma^{\mu} U_{\alpha j} \nu_{jL}\right) W^{-}_{\mu} \; + \; \text{diag. mass terms} \; + \; h.c.$$

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Assume, at time $t = 0$ and location $\vec{x} = 0$, a flavour eigenstate

$$|\nu(0,0)\rangle = |\nu_\alpha\rangle = \sum_i U_{\alpha j}^* |\nu_j\rangle$$

is produced. At time t and position $\vec{x}$, it has evolved into

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Oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \left| \left\langle \nu_\beta | \nu(t,\vec{x}) \right\rangle \right|^2 = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i(E_j - E_k)t + i(\vec{p}_j - \vec{p}_k)\vec{x}}$$

Problems with the textbook derivation

- In general, neither the equal energy assumption nor the equal momentum assumption is physically justified because both violate energy-momentum conservation in the production and detection processes.

R. G. Winter, Lett. Nuovo Cim. **30** (1981) 101
C. Giunti, W. Kim, Found. Phys. Lett. **14** (2001) 213, hep-ph/0011072
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Energy-momentum conservation for emission of mass eigenstate $|\nu_i\rangle$:

$$E_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\ \mathrm{MeV}$.

To first order in m_i^2:

$$E_i \simeq E + \xi \frac{m_i^2}{2E}\,, \qquad p_i \simeq E - (1-\xi)\frac{m_i^2}{2E}\,, \qquad \xi \approx \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

Oscillation formula for neutrino wave packets

Assume Gaussian wave packets:

$$|\nu_\alpha(x,t)\rangle = \frac{1}{(2\pi\sigma_{pS}^2)^{1/4}} \sum_j U_{\alpha j}^* \int \frac{dp}{\sqrt{2\pi}}\, e^{-(p-p_{jS})^2/4\sigma_{pS}^2}\, e^{-i\sqrt{p^2+m_j^2}\,t+ipx} |\nu_j\rangle$$

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Oscillation formula:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[-2\pi i \frac{L}{L_{jk}^{\rm osc}} - \left(\frac{L}{L_{jk}^{\rm coh}}\right)^2 - 2\pi^2\xi^2 \left(\frac{1}{2\sigma_p L_{jk}^{\rm osc}}\right)^2 \right]$$

C. Giunti, C. W. Kim, U. W. Lee, Phys. Rev. **D44** (1991) 3635; C. Giunti, C. W. Kim, Phys. Lett. **B274** (1992) 87

K. Kiers, S. Nussinov, N. Weiss, Phys. Rev. **D53** (1996) 537, hep-ph/9506271

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with

$L_{jk}^{\rm osc} = 4\pi E/\Delta m_{jk}^2$	Oscillation length
$L_{jk}^{\rm coh} = 2\sqrt{2}E^2/\sigma_p|\Delta m_{jk}^2|$	Coherence length
E	Energy that a massless neutrino would have
ξ	quantifies the deviation from E (tiny for Mössbauer neutrinos)
σ_p	Effective wave packet width (tiny for Mössbauer neutrinos)

Results from the wave packet treatment

- Decoherence term

$$\exp\left[-\frac{L}{L_{jk}^{\mathrm{coh}}}\right]$$

cannot inhibit oscillations because

$$L_{jk}^{\mathrm{coh}}/L_{jk}^{\mathrm{osc}} \sim E/\sigma_p \sim 10^{15}.$$

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$$\exp\left[-2\pi^2\xi^2\left(\frac{1}{2\sigma_p L_{jk}^{\rm osc}}\right)^2\right]$$

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⇒ Our expectation is confirmed:

$$P(\nu_\alpha \to \nu_\beta) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i\frac{\Delta m_{jk}^2 L}{2E}}.$$

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We are looking for a formalism, in which these quantities are automatically determined from the properties of the source and the detector.

The Grimus-Stockinger theorem

Let $\psi(\vec{p})$ be a three times continuously differentiable function on $\mathbb{R}^3$, such that ψ itself and all its first and second derivatives decrease at least like $1/|\vec{p}|^2$ for $|\vec{p}| \to \infty$. Then, for any real number $A > 0$,

$$\int d^3p \, \frac{\psi(\vec{p}) \, e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}| \to \infty} -\frac{2\pi^2}{L} \psi(\sqrt{A}\tfrac{\vec{L}}{L}) e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}}).$$

$\Rightarrow$ Quantification of requirement of on-shellness for large $L = |\vec{L}|$.

Amplitude for broadening by natural line width

Take into account the instability of ^{3}H in the source and the detector.

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1 \int_0^T dt_1 \int d^3x_2 \int_0^T dt_2 \left(\frac{m_{\rm H}\omega_{{\rm H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm H}\omega_{{\rm H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{{\rm H},S}t_1} \\
\cdot \left(\frac{m_{\rm He}\omega_{{\rm He},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm He}\omega_{{\rm He},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{+iE_{{\rm He},S}t_1} \\
\cdot \left(\frac{m_{\rm He}\omega_{{\rm He},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm He}\omega_{{\rm He},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{-iE_{{\rm He},D}t_2} \\
\cdot \left(\frac{m_{\rm H}\omega_{{\rm H},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm H}\omega_{{\rm H},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{+iE_{{\rm H},D}t_2} \\
\cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu *} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)} \\
\cdot \bar{u}_{e,S}\gamma_\mu \frac{1-\gamma^5}{2} \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \frac{1+\gamma^5}{2}\gamma_\nu u_{e,D}
\end{aligned}
$$

Amplitude for broadening by natural line width

Take into account the instability of ^{3}H in the source and the detector.

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1 \int_0^T dt_1 \int d^3x_2 \int_0^T dt_2 \left(\frac{m_{\rm H}\omega_{{\rm H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm H}\omega_{{\rm H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{{\rm H},S}t_1 - \frac{1}{2}\gamma t_1} \\
\cdot \left(\frac{m_{\rm He}\omega_{{\rm He},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm He}\omega_{{\rm He},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{+iE_{{\rm He},S}t_1} \\
\cdot \left(\frac{m_{\rm He}\omega_{{\rm He},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm He}\omega_{{\rm He},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{-iE_{{\rm He},D}t_2} \\
\cdot \left(\frac{m_{\rm H}\omega_{{\rm H},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\tfrac{1}{2} m_{\rm H}\omega_{{\rm H},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{+iE_{{\rm H},D}t_2 - \frac{1}{2}\gamma(T - t_2)} \\
\cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu *} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2 - t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)} \\
\cdot \bar{u}_{e,S}\gamma_\mu \frac{1-\gamma^5}{2} \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \frac{1+\gamma^5}{2}\gamma_\nu u_{e,D}
\end{aligned}
$$

Probability for broadening by natural line width

$$\mathcal{P} \propto \sum_{j,k} \theta(T_{jk})\, |U_{ej}|^2 |U_{ek}|^2$$

$$\cdot \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right] e^{i\left(\sqrt{\bar{E}^2 - m_j^2} - \sqrt{\bar{E}^2 - m_k^2}\right)L}$$

$$\cdot\, e^{-\gamma T_{jk}}\, e^{-L/L_{jk}^{\rm coh}}\, \frac{\sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j})\right] \sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k})\right]}{(E_S - E_D)^2}$$

$$T_{jk} = \min\left(T - \frac{L}{v_j},\, T - \frac{L}{v_k}\right)$$

$$L_{jk}^{\rm coh} = \frac{4\bar{E}^2}{\gamma |\Delta m_{jk}^2|}$$