Abstract submitted for the 12th International Congress of Logic, Methodology and Philosophy of Science (Oviedo, Spain, August 2003)

Section A.2 Philosophical Logic

DEFINITIONAL REFLECTION AND CIRCULAR REASONING

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The theory of definitional reflection provides a novel framework for studying logical features of circular, and especially paradoxical reasoning. Definitional reflection originated from reading clauses for atoms as definitions, thereby extending ideas concerning elimination rules in natural deduction [2, 3]. In the simplest (propositional) case, given a definition for an atom a of the form $\mathbb{D}: \begin{cases} a \Leftarrow \Delta_1 \\ \vdots \\ a \Leftarrow \Delta_n \end{cases}$, the rule of definitional reflection $(\mathbb{D}\vdash)\dfrac{\{\Gamma, \Delta_i \vdash C\}_i}{\Gamma, a \vdash C}$ is associated with a as a left introduction rule. If individual variables are present, and for computational purposes, the rule becomes more complicated [3, 5]. $(\mathbb{D}\vdash)$ is considered as introducing an atomic assumption a according to its definitional meaning given by $\mathbb{D}$. This is the *specific* way of introducing a as an assumption, which is distinguished from the *unspecific* way by means of an initial sequent $(a \vdash a)$. As in logic programming, $\mathbb{D}$ may contain arbitrary atoms, even a itself, without any well-foundedness requirement. Unlike definite Horn clause programming, the definientia Δ_i of a are not restricted to lists of atoms but may include, e.g., implications. This enables us to study, besides circular reasoning based on clauses like $a \Leftarrow a$, also paradoxical reasoning using clauses like $a \Leftarrow \neg a$ (i.e., $a \Leftarrow (a \to \bot)$). Considering the definition $\mathbb{D} := \{a \Leftarrow \neg a\}$, which may be regarded as an "abridged" form of a logical or set-theoretical paradox, we can distinguish three possible strategies, each of which blocks the derivation of absurdity $(\vdash \bot)$.

(1) We expect a derivation of absurdity to be direct (i.e., normal or without cuts). There is no such derivation, as all derivations of absurdity we can produce from $\mathbb{D}$ are indirect. This was discovered by Hallnäs [2] and is related to Ekman's paradox [1].

(2) We allow for assumptions to be introduced in an unspecific way only if no specific way of introducing them is available, i.e., if there is no appropriate definitional clause in $\mathbb{D}$. This corresponds to the requirement often made in the sequent calculus that initial sequents must be atomic. (Note that "atomic" in the logical sense corresponds here, where we are only dealing with atoms, to the fact that no definitional clause is available.) This idea is due to Kreuger [4, 6].

(3) We prohibit the identification of assumptions of the same shape but of a different kind (i.e., of assumptions introduced in an unspecific way vs. assumptions introduced by definitional reflection). This can be done by globally forbidding contraction, corresponding to the dealing with paradoxes in the tradition of BCK logic (Fitch, Curry, Ackermann, Grishin), or, preferably, by a more sophisticated procedure which keeps track of the origin of assumptions.

References

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