Nicholas Rescher on Greek Philosophy and the Syllogism

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1. THE DISCOVERY OF RATIONALITY

Greek philosophy has long fascinated philosophers and historians of philosophy alike. Here we will consider not a particular phase of the development of philosophy (or science) but its very foundation, at least in its European sense. Closely related to this is the idea that not only did a particular form of philosophy (and science) come into being (like, say, European philosophy), but that philosophy itself emerged according to its concept. Hence, if we identify the beginning of Greek thought with the beginning of reason, it becomes immediately clear that, in the broadest sense, terminological and methodological insights play a decisive role.[1]

Among these insights is the discovery of the possibility of theoretical propositions and proofs (Thales). In fact, it is both the geometrical (theoretical) propositions of Thales, which are copied by the general propositions of pre-Socratic philosophy, as well as the discovery of proof, later extended with the help of logical instruments (within the framework of axiomatic conceptions), which provide the first example of philosophical, i.e., rational thought and earned Greek thinking in general the validity that enabled it to step out of the shadow of myth once and for all.

Plato, for example, based his theoretical philosophy on the insights of geometry and introduces logic as a philosophically fundamental discipline in the framework of his epistemological insights in the *Theaetetus*, his linguistic investigations in the *Cratylus*, and his theory of truth and falsehood in the *Sophist*. Aristotle outlines his theory of a demonstrative science, and thereby his model of exact science, with respect to the axiomatic structure of geometry already under development, and with his syllogistics he establishes logic as a theory of justification in the narrower sense. However, in contrast to Plato's narrower definition of philosophy, which in terms of its subjects remains oriented on the geometric idea of ideality, Aristotle resumes the physical considerations of the pre-Socratic, especially Ionic, tra-

ditions. It was only through Aristotle that this tradition became part of the prehistory of philosophy, and cosmology the roots of philosophy in the Greek and modern sense.

Moreover, we have become used to understanding what we think *about* philosophical (and scientific) knowledge, by thinking *in* the language of Greek philosophy. Our concept of philosophical (and scientific) thought, as well as our concept of philosophical (and scientific) rationality both have a distinctly Greek history. Greek thought catches up with us not only historically—as an integral part of our cultural and historical consciousness—but also systematically: the *form* of our knowledge and our questioning is determined by rationality, the beginning of which we describe with Greek thought.

A consequence of this is that we cannot think of ourselves outside (the form) of Greek thought. And this also means that we cannot reflect the origin of this thought from the outside, as distant and disinterested observers, but only from the inside, as a part of a cause-effect relation. Methodologically (and epistemologically) formulated: Our reconstructions of the Greek origin of philosophical (and scientific) thought and of the rationality which realizes itself within this thought, are unavoidably determined by those (conceptual) orientations that we borrow from what we are reconstructing.

This need not be circular nor, methodologically speaking, 'historicist'. In essence, the point is to understand that in (the development of) Greek philosophy and science, an essential part of our philosophical and scientific self-understanding is also enclosed within it. For if it is true that Greek thought catches up with us both historically and systematically within the framework of the questions posed here, and that our form of thinking is determined by its Greek form, then we are investigating ourselves when we inquire into Greek thought. This is not only relevant insofar as we do philosophy (and science), but also in that we generally identify efforts of rational orientations with the essential idea of man. In this sense, as members of a rational culture, we are still Greek, and in this sense there exists no alternative to Greek thought and the concepts of reason and rationality, which came into the world through Greek thought. To use the language of Greek philosophy: *The beginning is the essence*—the Greek origin of philosophical and scientific thought, and its concepts of reason and rationality, are its essence.

It is, therefore, unsurprising that no philosopher can afford not to reflect on Greek philosophy, and that even the major philosophers continue to return to it again and again when posing questions, offering explanations, re-

constructing, and participating in other forms of philosophical debate. Nicholas Rescher is no exception.

2. COSMOS AND LOGOS

The importance and unique status of Nicholas Rescher in modern philosophy is plain to see in the number of his voluminous books, such as *A System of Pragmatic Idealism* in three volumes (1992-1994) and *Philosophical Reasoning* (2001). In comparison, his numerous shorter works are often overlooked, but it seems that these greatly complement the picture of his great contribution to philosophy, which is nonetheless impressive in and of itself. These shorter works often seem to be just preliminary studies for a more comprehensive work. But, this impression is false, and does not do justice to these shorter works, which include books such as *The Riddle of Existence* (1984) and *Epistemetrics* (2006), and also other books such as *Galen and the Syllogism* (1966) and the collection *Cosmos and Logos: Studies in Greek Philosophy* (2005). *Galen and the Syllogism* offers a discussion of Galen and the fourth figure of the syllogism in the light of new data from Arabic sources, while *Cosmos and Logos* deals with various aspects of mainly pre-Socratic philosophy. In this collection, Rescher summarizes his evaluation of Greek philosophy. This will here be shown by a perusal through the contributions summarized in this volume and in the analysis of his important earlier contribution to Galen and the history of logic.

In the studies found in *Cosmos and Logos*, we find masterstrokes of an interpretation which is argumentative, and which draws simultaneously on well-chosen source texts. This results in a high level of understanding that intentionally avoids much of the common literature analysis that normally suffocates an original approach. Here, Rescher follows two principles.

The first of these principles, which is explicitly stated, is that of an *interpretative restoration* of a theory that directs historical analyses, particularly in case the textual basis and the tradition is poor.[2] It corresponds to a concept of *reconstruction*, which can be defined as follows: A reconstruction is given or can be considered as successful or adequate, if a construction K' not only reproduces K correctly in all its essential parts but also, at the same time, fulfils the intentions pursued by K better (at least not worse) than K[3]—and this in Rescher's sense of supplementing information that makes the reconstructed argument more explicit.

The second of these principles, which is implicitly expressed, says that *historical* clarity (obtained from historical reconstruction) presupposes *systematic* clarity. That is, where there is no systematic clarity, nothing is seen, not even when reading texts. This can be rephrased in the following way using a well-known formula of Kant[4]: historical ideas (*Anschauungen*) without systematical knowledge (*Begriffe*) are blind—thereby taking into consideration the other side of the formula that concepts without contents are empty, which means here that a systematical interest needs a substantial basis, in this case philosophical thought in its historical development. Genesis and validity are tied together—also in Rescher's thought.

Central to the studies in *Cosmos and Logos* we accordingly find not the author and his work, but instead terminological constellations and methodological conceptions, which—as the following analysis will show—constitute the entire thought style of a philosophical era: of Greek philosophy in its essential cosmological and epistemological components, and simultaneously, for the most part, determine the further (philosophical and scientific) development. Examples are the concepts of *archē* (as primary matter and as principle), of opposition, of thought experimentation, of scepticism, of rational choice, and of logical reasoning.

3. FROM EVOLUTIONARY THOUGHT TO LOGICAL ANALYSIS

The first study is dedicated to cosmic evolution in Anaximander, and appeared before Charles H. Kahn's important book on the Greek philosopher[5] (the latter confirmed the former rather than rendering it outdated). The analysis is based on the assumption that Anaximander had an evolutionary concept of the development of the cosmos. The starting point here is the concept of *apeiron*, which is described and explained by means of Greek sources (Simplicius and others), with the result that the *apeiron* "lacks any quantitative as well as any qualitative definition"[6]. The correct conclusion is that "Anaximander's *apeiron* is limitless or boundless, but not literally infinite."[7] This, again, could serve as an example of the Aristotelian concept of *hypokeimenon*, which also Simplicius employs in his description of Anaximander's position.[8]

A *spatial* interpretation of the *apeiron*, which existed early on and which seems also to be shared by Rescher in his presentation of the cosmogonic process[9], leads, however, to more difficulties than solutions insofar as the ancient ideas of a material primary substance are again thrown into relief (how can something, which is itself not spatial, limit something

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that is?). An analysis of principles in the Aristotelian sense, which, in a way, is already present in Anaximander, again becomes an analysis of matter. However, it is still fascinating just how well Rescher succeeds in describing the evolution of Anaximander's cosmos (cosmogony) and its structures (cosmology) in his recourse to material and spatial ideas.

More importantly, though, as far as the systematic interest of philosophy in its own history (as well as the history of science) is concerned, is that the concept of scientific explanation[10] gains high profile with Anaximander, since here, for the first time, the explanation of natural phenomena, i.e., astronomical and meterological phenomena, succeeds, and succeeds in the form of a strictly *naturalistic* account ("the founder of scientific cosmogony"[11]). It should be added—diverging from Rescher's account—that this might also be valid in the sense of an analysis of principles when one interprets the concept of the *apeiron* not spatially, but in the sense of the Aristotelian concept of principle (*archē*)—whereas Aristotle curiously ignores Anaximander's conception in his interpretation of the pre-Socratics[12] which is oriented on the concept of the *archē*.

The analysis of the concept of *contrastive opposition*, on the other hand, is treated much differently. This concept can be held as a central element not only of pre-Socratic philosophy, but also of Greek philosophy as a whole, including Plato and Aristotle: the opposite determination of objects (through the determination of opposite qualities) and their balance.

In a precise manner Rescher presents various conceptions from Pythagoras through Aristotle, e.g., Heraclitus' theory of the unity of opposites, together with the claim that the development of a theory of opposites demonstrates a natural unfolding of a key idea "through substantively interconnected stages"[13]. This, however, would be too bold of a claim. For one, these conceptions are too different, and they also refer to a wide range of physical, arithmetic, ontological and logical objects. The idea seems plausible that processes can be explained through the existence of opposites and their balance (a central motif in Greek thought) within theoretical as well as practical philosophy. This idea was supported by both empirical and theoretical evidence, whereas the logical meaning of the concept of the opposite used here, namely as the opposition between statements, when one and the same object is assigned two contradictory determinations, was not clarified until much later, probably not until Plato, namely within the framework of his theory of truth in the *Sophist*.

The investigation into *thought experimentation* in pre-Socratic philosophy leads directly into methodology. Thought experimentation is seen by

Rescher to be a cognitive procedure which can be defined as follows: "A 'thought experiment' is an attempt to draw instruction from a process of hypothetical reasoning that proceeds by eliciting the consequences of an hypothesis which, for aught that one actually knows to the contrary, may well be false. It consists in reasoning from a supposition that is not (or not yet) accepted as true—and perhaps is even known to be false—but is assumed provisionally in the interests of making a point or establishing a conclusion."[14] With this very clear definition, Rescher, using examples from pre-Socratic argumentation, shows that an important instrument of generating ideas is at hand, and that it consists at the same time of "different styles of argumentation and reasoning"[15], which from the very beginning have determined the development of philosophical and scientific rationality. One after the other, Rescher proves, in concise analyses, that in pre-Socratic philosophy six various forms of thought experiments can be identified and reconstructed: explanatory conjectures (Thales), negatively demonstrative reasoning (Anaximander), *reductio ad absurdum* (the Pythagoreans), sceptical thought experimentation (Xenophanes), and analogical thought experimentation (Heraclitus). This last model, value dominance argumentation (Heraclitus), however, reinforces the fact that thought experimentation is a "cognitive instrument of substantial value"[16]. This procedure is, at the same time, typical of Rescher's philosophical approach: clarity in the systematic approach leads to clarity in the philosophical interpretation. This is something that the so called hermeneutic philosophy still has to learn.

On a much different level, Rescher deals with sophistic and sceptical streams of Greek philosophy. He attempts the rehabilitation of the sophists against Plato's verdict that—with very few exceptions, such as Hegel—greatly influenced how the sophists had been viewed within the history of philosophy. The chapter opens with the sceptical tropes of Aenesidemus and begins anew with a brief definition: "They were *apories*, that is, paradoxical considerations designed to manifest the infeasibility of obtaining secure knowledge about the world. For such knowledge is supposed to be objective and impersonal—uniformly true and valid for all people—whereas the tropes indicate that such solidity is unrealizable in matters relating to this world of ours. It thus emerges that knowledge of reality is unattainable."[17] This definition allows us to describe a sceptical process which follows, on the one hand, Protagoras, i.e., his sceptically interpreted maxim ("man is the measure of all things"[18]), while following, on the other hand, the principles of perspectivity (*isostheneia*, "the equivalency of dif-

ferent views"), of the plausibility or similarity with truth (*ta eikota*, "the likely-seeming") and of common sense (*epochē*, "the suspension of judgement and beliefflessness"). At the same time, scepticism and sophistry are clearly differentiated both systematically and historically: "what the Sceptics urged was a mental disengagement (*ataraxia*), an *epochē-geared detachment from judgmental thought in favor of living by 'appearances.'* By contrast, what the Sophists proposed was: *a commitment through interests and the requisites of effective praxis as mediated through our personal sense of desirability and value.*"[19] With this in mind, the way towards a more sympathetic understanding of sophist thought is open for Rescher. While the Sophists anticipate the anti-cognitivism of the Greek Sceptics, they at the same time avoid the cognitive nihilism of the Sceptics. Short commentaries on the *nomos-physis* distinction and its linguistic analysis in Plato's *Cratylus* make this clear, whilst at the same time the epistemological basis is layed deeper. Here, Rescher is again led by a systematic interest, namely to preserve systematic thought from its own inborn naïvité, as expressed in epistemological dreams of omnipotence.

The last two studies in this volume are dedicated again to logical and methodological themes. The chapter entitled "Anaximander, Aristotle and 'Buridan's Ass'" takes up the issue of the problem of choice without preference, as well as the suggestion that this problem has its roots in very different historical contexts: in the problem of physical symmetries (Anaximander, Plato, Aristotle), in the problem of how to explain God's choices to reason and to human rationalization (Ghazālī, Averroës), and in the problem of man's freedom of the will (Thomas Aquinas and others). Again, Rescher is not concerned about historical information as such, but instead about the systematic issue of a reasoned choice among equivalent alternatives, as relates to the difference between *reasons* and inclining *motifs*. The historical examples are well-chosen—these reveal the greater context in which this problem occurs in the history of philosophy—and simultaneously contribute significantly to the theory of rational choice, including the idea of randomness.

The final study ("Aristotle on Ecthesis and Apodeictic Syllogisms") rounds off the volume with a small masterpiece of logic. The focus here is on the method of *ecthesis*, which in Aristotle's logic is presented as a third possibility, in addition to conversion and *reductio ad absurdum*, for reducing syllogisms back to those of the first figure. The question arises as to how far the principle of modal *ecthesis*, required in the proof of the modi *baroco* and *bocardo* with necessary premisses and necessary conclusion, is

based on strictly logical considerations. This question is answered with reference to the fact that they are primarily metaphysical considerations, which here determine the argumentation and Aristotelian modal logic as a whole. Nonetheless, it is demonstrated that modal logic suffices for the same systematic demands as formal logic without modalities. This will be later on demonstrated in more detail (Section 6). A comparable task confronts Rescher in his short book on Galen and the syllogism.

4. GALEN AND THE FOURTH FIGURE

In his *Prior Analytics*, Aristotle distinguished only three figures of categorical syllogisms. Based on this tripartition, the syllogistic moods of the fourth figure have often been treated as 'indirect' moods of the first figure with the terms in the conclusion interchanged. The introduction of the fourth figure as an inference schema in its own right was credited to Galen for many centuries, and was sometimes even called the Galenian figure.[20] This attribution was called into question by Łukasiewicz[21], and many historians of logic followed him in that respect. Rescher, in his book *Galen and the Syllogism*[22], challenges this claim and argues that the attribution of the fourth figure to Galen is correct after all. To support this claim, he presents new evidence by providing an edition of a treatise by the Baghdad logician, physician and scientist Ibn al-Sarī, also known as Ibn al-Ṣalāḥ (ca. 1090-1153)[23] on the fourth figure of the syllogism. In this treatise, al-Ṣalāḥ not only defends the fourth figure, but also refers to (today no longer extant) sources available to him, which credit it to Galen. The (Arabic) edition and annotated translation of an Istanbul manuscript of this treatise—copied in 1229 from a valid source and in the modern Western scientific community known to exist since at least the 1930s[24]—is a masterpiece of philological and historical scholarship.[25] The ability to read and edit Arabic manuscripts is not normally something one would expect of a 'universal' philosopher but only of a specialist.[26]

The problem with ascribing to Galen the authorship of the fourth figure is that this claim rests only on indirect evidence. The extant writings of Galen tell us nothing directly about this issue. On the contrary, in his *Eisagogē dialektikē* (the only original Greek logic text known to have survived), he speaks of only three figures in the usual Aristotelian way.[27] Therefore, in order not to blame him for inconsistency, Rescher has to explain why Galen can both speak of three and of four figures arising from categorical syllogisms with two premisses. In fact, this should have (at

least implicitly) been undertaken within the tradition that credited the fourth figure to Galen. This makes it necessary for Rescher to discuss systematically the very concept of a syllogism. In doing so, he ends up with a book that, in addition to its historical and philological parts, contains a thorough examination of the foundations of syllogistics. It deals with three issues:

(1) the edition and translation of al-Ṣalāḥ's treatise,

(2) the concept of a syllogism and the significance of the fourth figure,

(3) the problem of crediting Galen with the authorship of the fourth figure.

Issues (1) and (2) can be studied independently of one another and of (3), whereas (3) relies on both (1) and (2).

Rescher's explanation as to why Galen may have considered both a three-figure and a four-figure treatment of syllogistics, is based on a distinction he draws between two conceptions of syllogisms, which lead to different views of syllogistic figures. Rescher distinguishes between (i) the Aristotelian two-premiss and (ii) the later two-premiss-cum-conclusion views of the syllogism, which may be described as follows.

(i) Given two premisses with one term shared between them (the middle term, M), we may pose the *syllogistic question*, asking what follows from them with respect to the two terms which are not shared by the premisses (the extreme terms, S and P). We would then distinguish between:

I. M occurring as the subject term in one premiss and as the predicate term in the other,

II. M occurring as the predicate term in both premisses,

III. M occurring as the subject term in both premisses.

Using x, y and z as variables for the syllogistic judgement forms a, i, e and o, this leads to the three syllogistic figures:

I.	II.	III.
$M \; x \; P$	$P \; x \; M$	$M \; x \; P$
$\underline{S \; y \; M}$	$\underline{S \; y \; M}$	$\underline{M \; y \; S}$
Z	Z	Z

The task of syllogistic theory would then be to establish which judgements Z of the form $S \, z \, P$ or $P \, z \, S$ are valid consequences of the respective premisses, depending on the judgement forms a, e, i or o assigned to x and y. Whether Z takes the form $S \, z \, P$ or $P \, z \, S$ is nothing that affects the distinction between the different forms. In fact, in figures II and III we can, without loss of generality, assume that Z takes the form $S \, z \, P$, as all conclusions of the form $P \, z \, S$ are obtained from the same figure by exchanging the order of the premisses, which is a logically insignificant operation. As the distinction between consequences of the form $S \, z \, P$ and $P \, z \, S$ is a subordinate matter even for the first figure, Aristotle is right to treat figure-I inferences with the conclusion $P \, z \, S$ as special (secondary, indirect) modes of the first figure. They are secondary, as the figure-I modes with $S \, z \, P$ as their conclusion are self-evident[28] and superior to all other forms of reasoning.

(ii) Given two premisses with a middle term M and extreme terms S and P, we may instead ask whether a conclusion *of a particular form* follows from them. In other words, we take the form of the conclusion to be part of the question. Therefore, in the case of the first figure, we do not just ask: "What follows from $M \, x \, P$ and $S \, y \, M$?" but rather, for a *given* judgement Z: "Does Z follow from $M \, x \, P$ and $S \, y \, M$?" As the form of Z is now relevant, this question must be divided from the very beginning into the two questions: "Does $S \, z \, P$ follow from $M \, x \, P$ and $S \, y \, M$?" and "Does $P \, z \, S$ follow from $M \, x \, P$ and $S \, y \, M$?". This leads to splitting up the first figure into two figures, the first (in the new sense) and the fourth:

I.	IV.
$M \; x \; P$	$P \; x \; M$
$\underline{S \; y \; M}$	$\underline{M \; y \; S}$
$S \; z \; P$	$S \; z \; P$

There is no difference with respect to the second and the third figure. Whereas figures I and IV are duals of each other with respect to permut-

ing premisses and converting the conclusion, figures II and III are self-dual. This view of the syllogistic figures has remained the standard.

From a modern point of view, the difference between (i) and (ii) is that between a *consequence-set view* ("What are the consequences of two given premisses?") and a *consequence-relation view* ("Which consequence relations between two given premisses and a given conclusion are valid?"). Aristotle's consequence-set view leads to a coarser classification than the medieval (and modern) consequence-relation view.

Rescher not only draws a conceptual distinction between these views, but also finds a terminological distinction which reflects this: A syllogistic figure, according to the three-figure view, is called a *schēma* by Aristotle (*syzygia* by Alexander of Aphrodisias), whereas according to the four-figure view, a figure is called *systasis* (*symplokē* by Alexander) and the corresponding syllogisms *technai*[29], with corresponding distinctions in Arabic. While only the first terminology is present in the extant texts of Galen (where he speaks of three figures), the second terminology occurs (in Greek) in his listings of his own works, and its Arabic counterpart is used in the Arabic texts which associate him with the fourth figure. Thus, Rescher makes the ascription of the fourth figure to Galen plausible by attributing to him the explicit awareness of these two views of the categorical syllogism.[30]

Not only those who, like Rescher, credit Galen with the fourth figure, have to explain how a three-figure and a four-figure view may coexist. Those who, like Łukasiewicz, deny Galen's authorship, must do so as well, in order to comply with the sources who claim his authorship (such as, e.g., Averroës), if they do not wish to simply disregard them as not trustworthy. Their way of arguing is that Galen, when speaking of a fourth figure, does not mean the fourth figure in the standard sense, but something different, e.g., syllogisms with more than two premisses.[31]

In any case, the ascription of the fourth figure to Galen is based on indirect sources and on the weight given to them, as none of the relevant papers by Galen are known to have survived, and even the text edited by Rescher does not unequivocally show this, as is demonstrated by Sabra's independently published commentary on this text.

4.1 AL-ṢALĀḤ'S TREATISE: RESCHER VS. SABRA

The very same Istanbul manuscript, which is the topic of Rescher's book, was published independently as a photographic reprint with a detailed commentary by the renowned historian of Arabic science A. I. Sabra.[32] It appeared a year earlier than Rescher's book and roughly at the same time as Rescher's preliminary report[33], but not before Rescher's book was completed.[34] In contradistinction to Rescher, Sabra follows Łukasiewicz in that Galen did not invent the fourth figure and interprets the references to Galen and the fourth figure in the introduction to al-Ṣalāḥ's treatise (which is the basis of Rescher's historical argument) accordingly.

In the introduction to this treatise[35], al-Ṣalāḥ makes the following four points which Rescher uses as arguments for Galen's authorship of the fourth figure:

(1) There is a commentary by Abū 'l-Faraj ibn al-Ṭayyib on the *Prior Analytics*, which criticizes Galen's introduction of the fourth figure.

(2) A treatise by Aḥmad ibn al-Ṭayyib al-Sarakhsī refers to a report mentioned to his master al-Kindī about the existence of a Syriac treatise of Galen dealing with the syllogistic figures. Al-Kindī explicitly rejects the fourth figure.

(3) There is said to be a treatise by al-Fārābī criticizing the fourth figure.

(4) There is a treatise by someone named Dinḥā the Priest, entitled *The Fourth Figure of Galen*, which is full of errors.

Sabra deals with these points in the following way:

(ad 1) This is acknowledged, but not given decisive evidence.[36]

(ad 2) This interpretation suffers from the vagueness of an Arabic expression. It is not clear from the text whether al-Kindī's rejection

of the fourth figure presupposes a claim by Galen against which it is directed.[37]

(ad 3) There is no surviving manuscript by al-Fārābī which supports this claim.[38] Actually, Rescher was hoping that the edition of further manuscripts of al-Fārābī might bring to light some reference to Galen and the fourth figure. This has not materialized so far.[39]

(ad 4) The identity of Dinḥā is not clear. Sabra does not further comment on this source. He would probably call into question its reliability, as apparently Dinḥā is not a logician.

It may be regretted that neither Rescher nor Sabra returned to the subject later. The merit of Sabra's paper is his extended commentary on the full text. Rescher essentially discusses its introduction, as this is the passage relevant to Galen, and considers the rest "largely a routine"[40]. However, the main text does contain some interesting observations on the fourth figure that may have fit into Rescher's general discussion on the merits of this figure (see below).

In general, it is somewhat surprising how little attention Rescher's and Sabra's discussions of al-Ṣalāḥ's treatise received by the experts in the field. In the philosophical literature, there is no reference to Sabra's paper whatsoever, and Rescher's is, for the most part, quoted vaguely in the style of "for a discussion of Galen and the origin of the fourth figure see Rescher". Even in the Arabistic literature there are references to Rescher and Sabra only here and then.[41] Given that there was so little general interest in the topic, the coincidence of these two projects which made al-Ṣalāḥ's treatise accessible to the public in 1965 is even more remarkable.

5. RESCHER AND THE FOURTH FIGURE

Though the consequence-relation-view of syllogistics, which gives the fourth figure its proper place, has dominated since medieval times, the fourth figure has nevertheless often been considered inferior to the other figures. Rescher discusses six arguments given in the literature to support this claim. He discards all of them and concludes with an argument of his own to establish it. Unfortunately, he does not deal with arguments which explicitly support the fourth figure (as compared to, say the second and

third), although that would have fit well into his book, as al-Ṣalāḥ's text is one that does exactly this. In the following, we discuss Rescher's reply to three traditional objections against the fourth figure (objections 2, 3 and 4 in his enumeration), as they are the most interesting from the systematic point of view. His replies to the three other objections, though historically highly relevant, are fairly obvious. Objections 2, 3 and 4 all deal with the deductive power of certain moods with respect to certain inference rules.

(1) *Objection 2* (stated in many traditional textbooks): The fourth figure is the first figure with an inverted conclusion. Rescher correctly replies that this is not true. One has to invert both the premisses and the conclusion to reach figure IV from figure I. Rescher correctly remarks that the true underlying reason for this objection is that only conversion is needed to deduce figure IV from figure I (which is traditionally considered the perfect figure), whereas without *reductio*, figures II and III cannot be obtained from I. So according to Rescher, figure IV may at best be considered deductively 'less independent' of figure I than figures II or III. However, even that depends on the fact that conversion is considered a weaker rule than *reductio*, as *reductio* alone suffices to generate figures II and III from I, so with respect to the number of rules required, figure IV is as near to figure I as are figures II and III. That conversion is weaker than *reductio* might actually be claimed as conversion can be justified using *reductio*, if a limiting case such as *S a S* is admitted as a premiss.[42]

(2) *Objection 3* (due to Leibniz): Figure IV is farther away from figure I than figures II and III, since in addition to *reductio*, conversion must be used to derive it. Rescher replies that Leibniz's assumption that conversion is less natural than *reductio*, is not warranted at all, and refers to the derivability of conversion from *reductio* (see above).

(3) *Objection 4* (credited by Rescher to Kneale & Kneale[43]): The fourth figure is inferior to the other figures, as *reductio* does not lead out of it. Rescher replies that this is no more than a fact. He does not mention that it is due to a distinguishing characteristic of the fourth figure, namely that the two occurrences of the three terms involved always occur in opposite positions—one on the left

and one on the right. This feature may actually give the fourth figure a superior quality (see Section 5.2).

Overall, according to Rescher, the objections to the fourth figure describe, if they are based on correct observations, mere *features* of this figure, without rendering it *inferior*.

Nevertheless, Rescher shares the opinion that the fourth figure is indeed inferior to the others, but on the basis of an argument which seems to him "to be the only one that posseses genuine force"[44]. It is based on the insufficient deductive power of the fourth figure, given the basic Aristotelian inferences to derive syllogistic moods from others (and in the end to reduce all valid moods to the moods of the first figure): *conversion* and *reductio* . Using these two inference principles, the valid moods of each of the figures I, II and III suffice to derive the valid moods of all four figures, whereas this is not true for the valid moods of figure IV. The moods *barbara*-I, *baroco*-II and *bocardo*-III cannot be reduced to the fourth figure by means of conversion and *reductio* alone. "It is yet another of the ironies of the history of logic that this (to our mind solitarily appropriate) ground for maintaining the inferiority of the fourth syllogistic figure is of our own devising and is nowhere to be found in the ramified literature of the subject, despite the innumerable discussions aimed at showing the inferiority of the fourth figure."[45] Rescher's systematic conclusion, then, is that the fourth figure, though indispensable, is "somewhat less central or fundamental"[46] than the others. Rescher attempts to put the traditional view of the inferiority of the fourth figure on a firm base by comparing its deductive power with that of the other figures.

5.1 AL-ṢALĀḤ'S ARGUMENTS IN FAVOUR OF FIGURE IV

As Rescher wants to dissociate his own objection to figure IV—which is not an argument against the figure as such, but against its deductive power—from other objections found in the literature, he does not discuss arguments *in favour* of figure IV. However, as they are the topic of the work edited, it is interesting to discuss al-Ṣalāḥ's arguments. Al-Ṣalāḥ makes at least three points: The first one is in favour of figure IV as a genuine syllogistic figure, the second and third are arguments for the superiority of figure IV over figures II and III. He even uses a new numbering system, calling figure IV the second, and the (traditional) figures II and III the third and fourth figures, respectively.[47] The status of figure I as the fig-

ure of the self-evident or self-justifying perfect syllogisms is not questioned.

(1) Al-Ṣalāḥ's central point in favour of figure IV is that it covers forms of reasoning that actually occur and as such must be addressed. Its reducibility to figure I is no argument, as figures II and III are reducible to figure I as well. Reducibility does not mean that the figure need not be considered. Al-Ṣalāḥ argues that once we make a distinction between the two premisses of a syllogism, we are automatically committed to distinguishing between the different positions of the middle term separating figures I and IV. To support this view, he gives a nice analogy: When we classify bodies into heavy and celestial, and correspondingly motion into straight and circular, and then associate heavy bodies with straight motion *towards the center*, then with qualifying straight motion into the two options *towards the center* and its opposite *from the center,* we are automatically committed to a corresponding division of the non-celestial bodies into heavy and light ones. This is a strong argument in favour of the consequence-relation view of syllogisms and thus for considering figure IV as a genuine form of reasoning.

(2) Figure IV is superior to figures II and III, as it allows for three judgement forms *e*, *i* and *o* in its conclusion, whereas figure II only allows for *e* and *o*, and figure III only for *i* and *o*. Figure I allows for all four forms *a*, *e*, *i*, and *o*. Thus the syllogistic figures are ranked according to the number of judgement forms possible in their conclusion. This is an argument concerning the *expressive* power, but not the *deductive* power, of figure IV. It is certainly a distinctive feature of this figure, but not necessarily one that makes this figure superior.

(3) As only conversion is required to pass from figure I to figure IV, figure IV is nearer to figure I than are figures II and III. This is closely related to the objection discussed at the beginning of Section 5. It relies on the assumption that conversion is a more natural form of reasoning than *reductio*. However, even if the distance of figure IV from figure I is smaller than that of figures II and III from I, we do not have a linear order. Figure IV does not lie in between figures I and II/III in strength.

5.2 *Merrill's Ignored Characteristic of Figure IV*

At the end of chapter II, which contains the systematic discussion of figure IV, Rescher refers to a paper by Merrill (1965) that establishes the superiority of figure IV over the other figures and thus calls into question Rescher's own stance as elaborated upon in this very chapter. This footnote was obviously added after the book was completed, perhaps even added after the book was in print. It says: "It is, moreover, undoubtedly the case that there are other systematic considerations—of perhaps a kind that seems artificial from the traditional approach to syllogistic logic—which militate favourably on behalf of the fourth figure. See, for example, D. D. Merrill [Reference to Merrill (1965)]." The rather moderate and indirect way of expressing himself: "*perhaps* a kind that *seems* artificial" shows that Rescher was aware that Merrill's paper challenges his basic systematic claim, namely that the fourth figure is inferior due to its lack of deductive power.

Reading Merrill's paper, it turns out that it puts forward an argument which supports a position opposite Rescher's (and at odds with the whole tradition of syllogistic theory). Quoting the paper and pointing to its significance demonstrates Rescher's intellectual honesty, as no one could or would have complained about not quoting a paper which appeared after the book was in print.[48] From today's point of view, this is even more remarkable. Merrill's paper, though highly relevant, short, clear, and to the point, has not been taken notice of at all, although *Mind* is one of the leading philosophical journals. As of now (December 2007), the *WebSci* and *Philosopher's Index* databases list no reference to this paper in any journal. Not even Thom in his extensive book on the syllogism[49] mentions it. We could not examine all of the books ever written about syllogistics, but Rescher's quotation is the only one we could find, although more then forty years have passed. It is unbelievable, how a most significant systematic contribution to syllogistics could remain unnoticed.[50]

Merrill's approach rests on the use of obversion rules, which were not introduced into the formal apparatus of syllogistics until the 19th century.[51] In this sense, Rescher is right to say that an approach based on obversion is "artificial from the traditional approach to syllogistic logic". On the other hand, as the idea of concept-negation which underlies obversion, is already present in Aristotle's work[52] (though not within his syllogistic formalism),

Rescher is also right in not repudiating such an approach outright. This is even more appropriate, as Rescher's own of method of assessing the significance of a syllogistic figure in terms of its strength to axiomatize syllogistics is not the most traditional way of treating syllogistic figures, either. A proper traditional approach would always put the allegedly self-justifying 'perfect' moods of figure I first, which Rescher does not.

As Merrill's paper is highly relevant, we sketch its content here and relate it to Rescher's approach. Suppose we have obversion rules at our disposal, which allow us to pass from a judgement to its contrary by changing a term to its complementary term, such as "All men are mortal" to "All men are not immortal" (i.e., "No man is immortal"), or "Some animals are mammals" to "Some animals are not non-mammals". Denoting the term complementary to P by P', and assuming that P'' yields P, obversion justifies the following equivalences:

$$S \, a \, P \Leftrightarrow S \, e \, P' \qquad S \, e \, P \Leftrightarrow S \, a \, P' \qquad S \, i \, P \Leftrightarrow S \, o \, P' \qquad S \, o \, P \Leftrightarrow S \, i \, P'$$

Call a *semi-syllogism* an inference figure, which looks like a syllogism, with the only difference being that any occurrence of a term S, M or P may be replaced by its complementary term S', M' or P', respectively. Note that we speak of *occurrences* of terms. This means that, in a semi-syllogism, a term may occur in one position in its plain form and in another in its complementary form. For example,

$$\frac{\begin{array}{l} P \ i \ M \\ S' \, a \, M' \end{array}}{S \ i \ P}$$

would be a semi-syllogism of the second figure (which is, of course, invalid).

Using obversion, any syllogism whatsoever can be converted into a semi-syllogism, in which only the judgement forms e and i occur. As e- and i-judgements are convertible, this semi-syllogism can be transformed into a semi-syllogism of any figure by conversion (where premises must be permuted if the conclusion is converted). Now only the fourth and no other figure has the property that by means of obversion, any *semi-syllogism* of a given figure can be transformed into a *syllogism* of the same figure. This is due to the fact that only in the fourth figure do the two occurrences of a term always occur in opposite positions (something on

which Leibniz's objection to the fourth figure is based, i.e. something which was considered a negative feature of figure IV, see (2) at the beginning of Section 5). This means, in particular, that one of the occurrences must occur in the predicate position and therefore obversion is permitted. This holds equally for valid and for invalid moods.

Consider, for example, *barbara*-I:

  M a P
  S a M
  S a P

Obversion of all three judgements gives

  M e P'
  S e M'
  S e P'

from which by conversion of both premisses we obtain the following semi-syllogism of the fourth figure

  P' e M
  M' e S
  S e P'

From this, by obversion of the first premiss, we obtain

  P' a M'
  M' e S
  S e P'

which is an instance of *calemes*-IV. If, in this example, we replace *a* and *e* with *o* and *i*, respectively, we obtain a reduction of the invalid mood *ooo*-I of the first figure to the invalid mood *oii*-IV of the fourth figure.

In this way, Merrill shows that the valid moods of the fourth figure axiomatize the valid moods of all other figures by means of conversion and obversion only. Moreover, the invalid moods of the fourth figure axiomatize the invalid moods of all other figures. Axiomatization of the invalid moods goes beyond the classical picture of syllogistics, although it is not

too far from it, since most reduction procedures studied in classical syllogistics are symmetrical and independent of the validity of the moods considered.[53] If obversion is considered an admissible way of reasoning, this shows that the fourth figure is not inferior, but indeed superior to figures I, II and III. Actually, for the reduction of the valid moods of any figure to those of IV, conversion restricted to the premises (*conv*) and obversion (*obv*) is needed. For the reduction of the valid moods of any figure to those of one of the figures I, II or III, we need *reductio* (*red*) plus unrestricted conversion (*convx*).[54] Rescher's claim that the fourth figure is inferior can only be upheld if *red+convx* is considered less elementary or less natural than *obv+conv*. This essentially amounts to the question of whether obversion is considered an elementary mode of inference. Merrill's paper demonstrates again that all claims based on the deductive power of syllogistic moods or figures strongly depend on the transformation rules assumed, and that the appropriate choice of such rules is not a trivial matter at all.

6. ECTHESIS, MODAL SYLLOGISTICS, AND THE METAPHYSICAL BASIS OF LOGIC

In the final paper of *Cosmos and Logos*[55], Rescher extends the principle of *ecthesis* introduced by Aristotle in his assertoric syllogistics, to the modal realm, something which by Aristotle is only sketched in a few lines of text, but not developed in detail.[56] Using this extension, he is able to explain certain features of this system which have been a stumbling block to many prominent authors dealing with the subject. *Ecthesis*, in the standard non-modal sense, turns a particular judgement into two universal judgements, namely

$S\,i\,P$	into	$N\,a\,S$ and $N\,a\,P$
$S\,o\,P$	into	$N\,a\,S$ and $N\,e\,P$

claiming that a term N can always be found. Actually, in the first case, for N we can choose the term $(S \cap P)$, whose extension is the intersection of the extensions of S and P, whereas in the second case, we can choose the term $(S \backslash P)$, whose extension is the difference of the extensions of S and P.[57] In the modal case, *ecthesis* turns an apodictic particular judgement into two apodictic universal judgements, namely

$\Box(S\,i\,P)$	into	$\Box(N\,a\,S)$ and $\Box(N\,a\,P)$

$$\Box(S \ o \ P) \qquad \text{into} \qquad \Box(N \ a \ S) \text{ and } \Box(N \ e \ P)$$

As in the assertoric case, this can be validated by assuming N to be $(S \cap P)$ and $(S \backslash P)$, respectively. However, Rescher wants to characterize N not just extensionally, but by means of a choice operator which from S and P selects a *natural kind*, for which the universal apodictic assertions hold true. This is then metaphysically interpreted as expressing Aristotle's *tenet* that "science, since it deals with the necessary, cannot but deal with the universal as well"[58], i.e., a necessary *particular* judgement like $\Box(S \ i \ P)$ or $\Box(S \ o \ P)$ becomes a *scientific* judgement by turning it into necessary *universal* judgements in the way indicated. This is, of course, a highly metaphysical claim linking logic with metaphysics. It cannot rest on logic alone, as the assumption that there is a *natural kind N* with the mentioned properties is not a purely logical postulate.

In order to be able to "construct"[59] such an N, Rescher introduces a bracket notation $[SP]$ for terms S and P such that, if all S are P, then all S are necessarily $[SP]$. Thus Rescher uses as an axiom the principle

$$(1) \quad \frac{S \ a \ P}{\Box(S \ a \ [SP])}$$

His second axiom is

$$(2) \quad \frac{\Box(S \ a \ P)}{\Box([QS] \ a \ P)} \ ^{60}$$

From (1) and (2) together with corresponding principles for *i*- and *e*-judgements

$$(1') \quad \frac{S \ i \ P}{\Box(S \ i \ [SP])} \qquad\qquad (2') \quad \frac{\Box(S \ e \ P)}{\Box([QS] \ e \ P)}$$

he is able to derive the valid moods of the first figure of Aristotle's modal syllogistics and show that the invalid moods are not derivable.[61] This includes the validity of the (infamous) *barbara LXL*

$$\frac{\Box(M \ a \ P)}{\frac{S \ a \ M}{\Box(S \ a \ P)}}$$

and the underivability of *barbara XLL*

$$M \, a \, P$$
$$\frac{\Box(S \, a \, M)}{\Box(S \, a \, P)}$$

Modal *ecthesis* is then axiomatized using the axiom

$$\text{(3)} \quad \frac{\Box(S \, i \, P)}{(\exists Q)(\Box([SQ] \, a \, S) \wedge \Box([SQ] \, a \, P))}$$

and analogously for *o*-judgements.[62] So the intuitive idea is that there is some predicate *Q* constituting a natural kind [*SQ*], such that from the particular modal judgement we obtain two universal modal judgements with the subject term [*SQ*]. By adding the standard reduction rules, this constitutes an axiomatization of the full system of Aristotle's modal syllogistics.

Though as a formal system this is impressive, its intuitive interpretation is not entirely clear. The *ecthesis* axiom (3) suggests that in [*SQ*] the *Q* denotes a property essential to all *S*'s, i.e., a *differentia specifica* singling out in a natural way those *S*'s that are also *P*'s. This view also underlies the reading of [*QS*] in (2): If in some possible world a [*QS*] did not have the property *S*, then we could not infer from $\Box(S \, a \, P)$ that it has the property *P*. On the other hand, the term [*SP*] in axiom (1) is interpreted differently, as here [*SP*] denotes the natural kind *N* within *P* which is specified in a way such as to make $\Box(S \, a \, N)$ deducible from the non-modal statement *S a P*. The reading of [*SP*] with *P* being essential to *S* would naturally validate the judgement $\Box([SP] \, a \, P)$ which, together with (1), makes it possible to derive $\Box(S \, a \, P)$ from *S a P*, thus trivializing modal logic. Though the system is formally consistent, Rescher's bracket terms are semantically ambiguous.[63]

This is not surprising, as in Aristotle's modal syllogistics the seemingly trivial judgement $\Box(S \, a \, S)$ cannot be valid. This is due to the fact that using *barbara LXL*, $\Box(S \, a \, S)$ trivializes modal logic.[64] In Rescher's axiomatization, this is obtained by applying (2) to the premiss $\Box(S \, a \, S)$, yielding the trivializing judgement $\Box([SP] \, a \, P)$ mentioned above.[65]

From this point of view, Rescher's axiomatization of modal syllogistics using bracket terms is only formally, but not semantically, significant. However, there is a way out of this in keeping with the essentialist interpretation that guides Rescher's formalization, though it means sacrificing

certain parts of Aristotelian modal syllogistics. We interpret "every S is necessarily P" not as the *de dicto* statement "it is necessary that every S is P" ($\Box(S\,a\,P)$, in modern terminology $\Box\forall x(Sx \to Px)$), but as the genuinely essentialist *de re* statement "every S has P as a necessary (= essential) property" ($S\,a\,\Box P$, in modern terminology $\forall x(Sx \to \Box Px)$). Then it is obvious that $S\,a\,\Box S$ is not universally valid, as not every S that we pick in the actual world need to be an S in every other possible world. The bracket term $[SP]$ now denotes a rigid concept[66] which in the actual world has the extension of $(S\cap P)$. Then every object which, in the actual world, has the property $[SP]$ (and thus the properties S and P), has the very same property $[SP]$ (but not always the properties S or Q) in every other possible world. Aristotle's basic metaphysical claim expressed in axiom (1) would then be that whenever $S\,a\,P$ holds in the actual world, there is some natural property $[SP]$, which every S (picked in the actual world) has in the actual as well as in every other world. Axiom (2) is validated likewise: If every S (picked in the actual world) has the property P in all possible worlds (i.e., $S\,a\,\Box P$ holds true), then every $[QS]$ (picked in the actual world), as being an S in the actual world, has the property P in all possible worlds.[67] Correspondingly, the validity of *barbara LXL* and the invalidity of *barbara XLL* result naturally.

Obviously, this view invalidates all parts of Aristotelian modal syllogistics that rest on conversion laws, as $S\,i\,\Box P$ does no longer imply $P\,i\,\Box S$. However, these parts of modal syllogistics can more easily be discarded[68] than Aristotle's general metaphysical views whose intimate relation to logic is pointed out by Rescher. Converting the title of a book by Michael Dummett[69], we may speak of the *metaphysical basis of logic* which is evident in Aristotle's modal syllogistics.

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NOTES

[1] See Mittelstrass (2003), 134-157.

[2] Rescher (2005), 1.

[3] Mittelstrass (1985), 92.

[4] *Critique of Pure Reason* B 75.

[5] Kahn (1960).

[6] Rescher (2005), 3.

[7] *Ibid*.

[8] *In Phys*. 24, 13f. (*VS* 12 A 9).

[9] Rescher (2005), 9.

[10] Rescher (2005), 24.

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[11]   Rescher (2005), 26.

[12]   *Met*. A3.983b6ff.

[13]   Rescher (2005), 44.

[14]   Rescher (2005), 47.

[15]   Rescher (2005), 60.

[16]   *Ibid*.

[17]   Rescher (2005), 63.

[18]   Plato, *Theaet*. 152a; Aristotle, *Met*. K6.1062b13-14.

[19]   Rescher (2005), 76.

[20]   This view was propagated by Averroës (see Sabra 1965b, 14).

[21]   Łukasiewicz (1957), § 14 (38-42).

[22]   Rescher (1966). For reviews of this book see Thomas (1967), Geach (1968), Pingree (1968), Jager (1970).

[23]   Al-Ṣalāḥ died in Damascus in A.H. 548 = A.D. 1153-54. See Sabra (1965b), 15.

[24]   See Sabra (1965b), 14-16.

[25]   It is a striking coincidence that the same manuscript was published independently at the same time by A. I. Sabra (1965b). See the next section.

[26]   Actually, this book is not Rescher's first publication of this kind—see Rescher (1963).

[27]   Rescher (1966), 3-4.

[28]   "Perfected through themselves" in Aristotle's terminology (*An. pr*. A5.26b26-35, 29b6-8).

[29]   Rescher (1966), 17-19.

[30]   Jager (1970) argues that the ancient Greek tradition after Galen should be taken into account in order to make this "admittedly indirect and circumstantial evidence" (Rescher 1966, 17) more conclusive.

[31]   Rescher (1966), 2. This was first claimed by Łukasiewicz (1957, 39), who refers to an anonymous scholium on Ammonius' commentary on the *Prior Analytics*.

NOTES

[32] Sabra (1965b). Sabra did his doctorate with K. R. Popper in London and later became Professor of the History of Arabic Science at Harvard University. Actually, he wrote a (not so favorable) Review of Rescher (1963), see Sabra (1965a).

[33] Rescher (1965).

[34] The preface of Rescher (1966) dates from February 1965. We owe the Sabra (1965b) reference to Thomas Piecha.

[35] Rescher (1966), 52-54; Sabra (1965b), 16-18.

[36] Sabra (1965b), 18.

[37] *Ibid.*

[38] Sabra (1965b), 18f.

[39] Lameer (1994, 1996); Street (1997).

[40] Rescher (1966), 50.

[41] For example, in Lameer (1994).

[42] Use datisi-III and cesare-II, which themselves can be obtained from figure I using *reductio*.

[43] Kneale & Kneale (1962), 101.

[44] Rescher (1966), 47.

[45] *Ibid.*

[46] Rescher (1966), 48.

[47] In the following, to avoid any confusion, we use the traditional numbering throughout.

[48] The preface dates from February 1965, and the first 1965 issue of *Mind* was available in the U.S. probably considerably later than this date.

[49] Thom (1981).

[50] In an e-mail, Professor Merrill confirmed to us that he knows of no other quotation beyond that of Rescher's.

[51] Menne (1984).

[52] *De Int*. 10.20a19-26.

NOTES

[53] Exceptions are *ecthesis* and rules which involve subalternation, such as *conversio per accidens*.

[54] Here, the "*x*" stands for the permutation of premises, which is needed when the conclusion is converted, as, for example, in the reduction of *bocardo*-III to *darii*-I.

[55] "Aristotle on Ecthesis and Apodeictic Syllogisms". This paper is a revised version of a paper co-authored by Z. Parks (Rescher & Parks 1971) entitled "A new approach to Aristotle's apodeictic syllogisms". Some of the revisions are substantial, so the serious student of Aristotelian modal syllogistics should consult the 1971 paper as well. Unfortunately, the 2005 edition also contains several serious misprints.

[56] *An. pr.* A9.30a6-14.

[57] We disregard the problem of existential import, which becomes relevant when reading these inference steps from right to left.

[58] Rescher (2005), 126.

[59] "Construction" here is understood in the metaphysical sense, not in any "constructivist" way of thinking.

[60] Rescher (2005), 117. In Rescher & Parks (1971, 680), this axiom is just presented as such, whereas in Rescher (2005) it is reduced to its nonmodal variant, assuming the (Aristotelian) principle that every assertoric inference yields a corresponding apodictic inference (which, in modern terminology, comes down to assuming the principles of necessitation and distribution).

[61] Rescher (2005), 117-120.

[62] Rescher (2005), 122f.

[63] This is reflected in the terminology used by Rescher and the different ways in which he describes the meaning of [*SP*]. He speaks of [*SP*] "to represent the *P*-<sub>species of *S*" (1971, 678; 2005, 115, text in angle brackets only in 2005), of [*SP*]'s as "specifically those *S*'s that are by nature *P*'s", as "those *S*'s which *must* be *P*'s in virtue of their being *S*'s (i.e. by conditional or relative necessitation)" (116, "must" emphasized in 2005), of [*SP*] "such as to validate <automatically> the <modality-strengthening> inference (1)" (text in angle brackets added 2005), as "the *P*-oriented subspecies of *S* (consisting of *S*'s that are *P*'s by virtue of being *S*'s)" (only 2005), of [*SP*]'s as "*P*'s with a relative necessity, subject to the condition of their being *S*'s" (only 1971), as "*S*'s-that-in-fact-are-*P*'s" (only 1971, footnote 4). Concerning □([*SP*] *a P*), Rescher just writes in 2005 that this is "unattainable in the circumstances" (Rescher 2005, footnote 3), without giving any further reason. (It is clear that it *must* be unattainable.) Rescher gives the impression of an author still struggling to make explicit what he has in mind. He vacillates between claiming that [*SP*] is a subspecies of *S* with *P* being a specifying term, and considering it a

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species that is specified in some other way, though it contains the same objects. Our proposal at the end of this section attempts to give [*SP*] a precise meaning.

64   As has often been stated, Aristotle is silent about that result, although he may have been aware of it. See McCall (1963), 50.

65   As a potential way out, the 1971 paper (Rescher & Parks 1971, footnote 2) contemplates an observation put forward by its coauthor in Parks (1972), where he argues that an identification of variables as in $\Box(S\,a\,S)$ is never considered by Aristotle. However, in the end, Rescher & Parks (1971, footnote 7) prefer accepting the invalidity of $\Box(S\,a\,S)$. This is also correct from the perspective of our reconstruction below. Changing the substitution rule appears *ad hoc* without systematic warrant.

66   Rigid in the sense that its extension is the same in all possible worlds. For simplicity, we assume the domain of objects to be constant throughout possible worlds.

67   This is now a direct semantic justification of axiom (2). Rescher's justification in the 2005 paper, which relies on the validity of the assertoric counterpart of (2), is no longer valid under the essentialist reading of universal apodictic judgements.— The idea of interpreting apodictic judgements as *de re* statements is not new. Its modern history goes back to Becker (1933, see also Nortmann 1996). The problems which apodictic judgements pose to the conversion laws (see below) were a central point in medieval discussions of modal syllogistics (Lagerlund 2000).

68   Namely by attributing them to a lack of distinction between *de re* and *de dicto* modalities.

69   Dummett (1991).

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