# Dialogical logic, old and new

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## 1 The dialogical framework

### 1.1 Four basic structural rules

**Structural rule 1 =** *starting rule*

A dialogue game starts with a player stating a thesis, *i.e.*, a complex proposition. This player is the Proponent, the other is the Opponent. This is move 0.

*Repetition ranks: each player, starting with the Opponent, chooses a repetition rank. This is the maximum number of times the player will be allowed to challenge the same statement. The Proponent should always choose at least a rank one higher than the Opponent.*

**Structural rule 2 =** *(intuitionistic) playing rule*

Each player in turn makes a move. After stating the thesis, each move is either a challenge addressed against a previous move, or a response to the last unanswered challenge. The moves made follow the particle rules.

**Structural rule 3 =** *socratic rule*

The Proponent cannot state an elementary proposition ($A$, $B$, etc.) if the Opponent has not previously stated it.

**Structural rule 4 =** *winning rule*

A dialogue stops when a player has no available move to make, neither challenge nor response. This player loses the dialogue game, the other wins.

### 1.2 The particle rules for propositional logic

| Rule | Statement | Challenge | Answer |
|---|---|---|---|
| Conjunction | $\mathbf{X}!\ (A \wedge B)$ | $\mathbf{Y}?_{\wedge 1}$ <br> $\mathbf{Y}?_{\wedge 2}$ | $\mathbf{X}!\ A$ <br> $\mathbf{X}!\ B$ |
| Disjunction | $\mathbf{X}!\ (A \vee B)$ | $\mathbf{Y}?_{\vee}$ | $\mathbf{X}!\ A$ <br> $\mathbf{X}!\ B$ |
| Implication | $\mathbf{X}!\ (A \to B)$ | $\mathbf{Y}!\ A$ | $\mathbf{X}!\ B$ |
| Negation | $\mathbf{X}!\ \neg A$ | $\mathbf{Y}!\ A$ | ✖ |

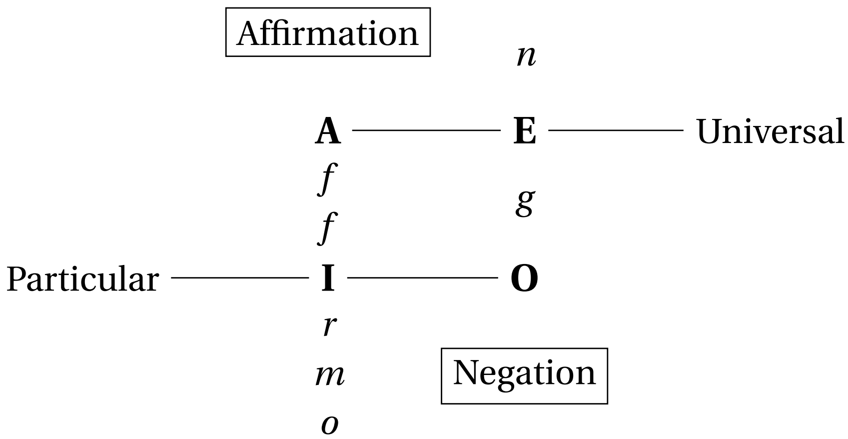
# 2 Aristotle’s logic

## 2.1 The Organon (logical treatises)

| | | |
|---|---|---|
| *Categories* | ***Prior Analytics*** | *Topics* |
| *De Interpretatione* | *Posterior Analytics* | *Sophistical Refutations* |

## 2.2 Syllogistic in the *Prior Analytics*

| | *First figure* | *Second figure* | *Third figure* |
|---|---|---|---|
| *Premise 1* | $P$ belongs to $M$ | $M$ belongs to $P$ | $P$ belongs to $M$ |
| *Premise 2* | $M$ belongs to $S$ | $M$ belongs to $S$ | $S$ belongs to $M$ |
| *Conclusion* | $P$ belongs to $S$ | $P$ belongs to $S$ | $P$ belongs to $S$ |




| FIRST FIGURE | SECOND FIGURE | THIRD FIGURE |
|---|---|---|
| *Barbara* | (*Baroco*) | (*Bocardo*) |
| *Celarent* | *Cesare*<br>*Camestres* | |
| *Darii* | | *Darapti*<br>*Disamis*<br>*Datisi* |
| *Ferio* | *Festino* | *Felapton*<br>*Ferison* |

## 2.3 The *dictum de omni et de nullo*

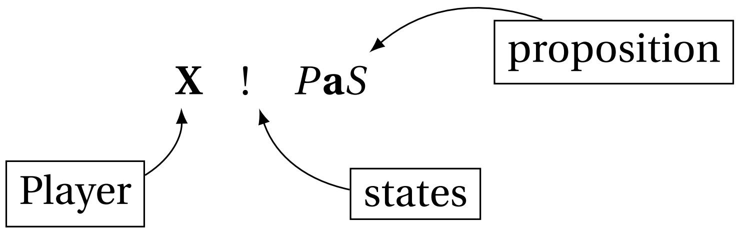
We use the expression «predicated of every» when none of the subject can be taken of which the other term cannot be said, and we use «predicated of none» likewise.

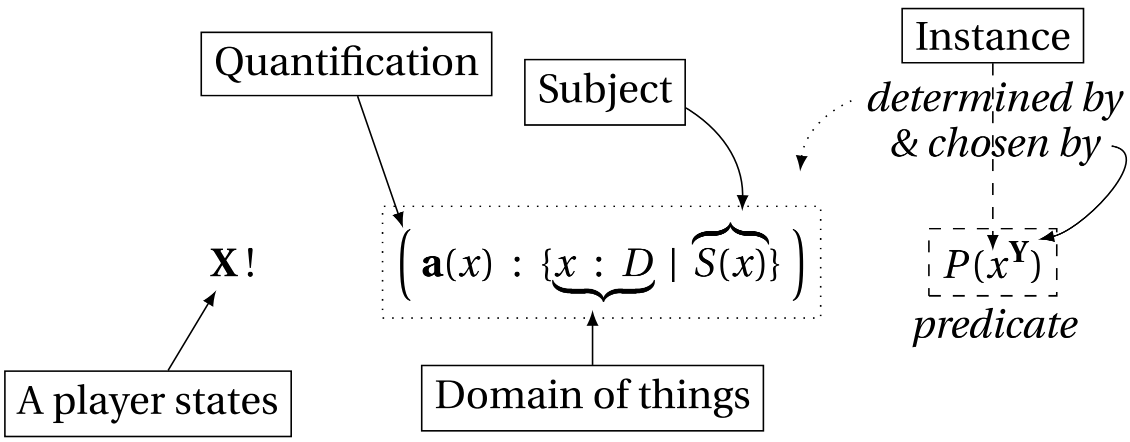
*Prior Analytics* I 1, 24b28–30; transl. Smith

# 3 Syllogistic in the dialogical framework

## 3.1 The notation

The player **X** states that $P$ is predicated of all the $S$s:




| NOTATION | |
|---|---|
| Traditional | Immanent Reasoning |
| $\mathbf{X}\,!\,P\mathbf{a}S$ | $\mathbf{X}\,!\,\big(\mathbf{a}(x) : \{x : D \mid S(x)\}\big)P(x^{\mathbf{Y}})$ |
| $\mathbf{X}\,!\,P\mathbf{e}S$ | $\mathbf{X}\,!\,\big(\mathbf{e}(x) : \{x : D \mid S(x)\}\big)P(x^{\mathbf{Y}})$ |
| $\mathbf{X}\,!\,P\mathbf{i}S$ | $\mathbf{X}\,!\,\big(\mathbf{i}(x) : \{x : D \mid S(x)\}\big)P(x^{\mathbf{X}})$ |
| $\mathbf{X}\,!\,P\mathbf{o}S$ | $\mathbf{X}\,!\,\big(\mathbf{o}(x) : \{x : D \mid S(x)\}\big)P(x^{\mathbf{X}})$ |

### 3.2 The particle rules

| Trad. | Statement | Challenge | Answer |
|---|---|---|---|
| $P\mathbf{a}S$ | $\mathbf{X}\,!\,\Big(\mathbf{a}(x) : \{x : D \mid S(x)\}\Big)P(x^{\mathbf{Y}})$ | $\mathbf{Y}\,!\,S(d)$ | $\mathbf{X}\,!\,P(d^{\mathbf{Y}})$ |
| $P\mathbf{i}S$ | $\mathbf{X}\,!\,\Big(\mathbf{i}(x) : \{x : D \mid S(x)\}\Big)P(x^{\mathbf{X}})$ | $\mathbf{Y}\,?_D$ | $\mathbf{X}\,!\,S(d)$ |
| | | $\mathbf{Y}\,!\,S(d^{\mathbf{X}})$ | $\mathbf{X}\,!\,P(d)$ |
| $P\mathbf{e}S$ | $\mathbf{X}\,!\,\Big(\mathbf{e}(x) : \{x : D \mid S(x)\}\Big)P(x^{\mathbf{Y}})$ | $\mathbf{Y}\,!\,S(d)$ | $\mathbf{X}\,!\,P(d^{\mathbf{Y}})^{\perp}$ |
| $P\mathbf{o}S$ | $\mathbf{X}\,!\,\Big(\mathbf{o}(x) : \{x : D \mid S(x)\}\Big)P(x^{\mathbf{X}})$ | $\mathbf{Y}\,?_D$ | $\mathbf{X}\,!\,S(d)$ |
| | | $\mathbf{Y}\,!\,S(d^{\mathbf{X}})$ | $\mathbf{X}\,!\,P(d)^{\perp}$ |
| Negation | $\mathbf{X}\,!\,P(d)^{\perp}$ | $\mathbf{Y}\,!\,P(d)$ | $\mathbf{X}\,\perp$ |

### 3.3 The structural rules

| # | Name | Structural rule |
|---|---|---|
| 1. | Starting rule | **P** states the thesis (move 0).<br>**O** states the premises; **P** states the conclusion (move 2). |
| 2. | Development rule | **O** & **P** take turns, challenging or answering. |
| 3. | Socratic rule | **P** may **not** state an **elementary proposition** unless backed by **internal reason** $you_i$. |
| 4. | Pragmatic coherence rule | Deals with a particular case: based on the recapitulation interpretation of syllogisms, allows **P** to ask **O** for an instance of subject. |
| 5. | Ending rule | Stating $\perp$ makes the player lose. Not being unable to move makes the player lose. |

## 3.4 Proofs

### 3.4.1 Barbara

$$\mathbf{P}\,!\;\underbrace{\Big(\mathbf{a}(x)\,:\,\{x:D|C(x)\}\Big)\;A(x^{\mathbf{O}})}_{A\mathbf{a}C}\;\Big[\underbrace{\Big(\mathbf{a}(x)\,:\,\{x:D|B(x)\}\Big)\;A(x^{\mathbf{P}}),}_{A\mathbf{a}B}\;\underbrace{\Big(\mathbf{a}(x)\,:\,\{x:D|C(x)\}\Big)}_{B\mathbf{a}C}$$

| | **Opponent** | | | **Proponent** | |
|---|---|---|---|---|---|
| | | | | $!\,A\mathbf{a}C\,[A\mathbf{a}B,\,B\mathbf{a}C]$ | 0 |
| 1.1<br>1.2 | $!\Big(\mathbf{a}(x):\{x:D\|B(x)\}\Big)A(x^{\mathbf{P}})$<br>$!\Big(\mathbf{a}(x):\{x:D\|C(x)\}\Big)B(x^{\mathbf{P}})$ | 0 | | $!\Big(\mathbf{a}(x):\{x:D\|C(x)\}\Big)A(x^{\mathbf{O}})$ | 2 |
| 3 | $!\,C(d)$ | 2 | | $you_7 : A(d^{\mathbf{O}})$ | 8 |
| 5 | $!\,B(d^{\mathbf{P}})$ | | 1.2 | $you_3 : C(d)$ | 4 |
| 7 | $!\,A(d^{\mathbf{P}})$ | | 1.1 | $you_5 : B(d)$ | 6 |

Proponent wins

### 3.4.2 e-conversion

| | **Opponent** | | | **Proponent** | |
|---|---|---|---|---|---|
| | | | | $!\,B\mathbf{e}A\,[A\mathbf{e}B]$ | 0 |
| 1 | $!\Big(\mathbf{e}(x):\{x:D\|B(x)\}\Big)A(x^{\mathbf{P}})$ | 0 | | $!\Big(\mathbf{e}(x):\{x:D\|A(x)\}\Big)B(x^{\mathbf{O}})$ | 2 |
| 3 | $!\,A(d)$ | 2 | | $!\,B(d^{\mathbf{O}})^{\perp}$ | 4 |
| 5 | $!\,B(d)$ | 4 | | | |
| 7 | $!\,A(d^{\mathbf{P}})^{\perp}$ | | 1 | $you_5 : B(d)$ | 6 |
| 9 | $\perp$ | | 7 | $you_3 : A(d)$ | 8 |

Proponent wins

### 3.4.3 invalid moods

| | **Opponent** | | | **Proponent** | |
|---|---|---|---|---|---|
| | | | | $!\,A\mathbf{a}C\,[A\mathbf{a}B,\,B\mathbf{e}C]$ | 0 |
| 1.1<br>1.2 | $!\Big(\mathbf{a}(x):\{x:D\|B(x)\}\Big)A(x^{\mathbf{P}})$<br>$!\Big(\mathbf{e}(x):\{x:D\|C(x)\}\Big)B(x^{\mathbf{P}})$ | 0 | | $!\Big(\mathbf{a}(x):\{x:D\|C(x)\}\Big)A(x^{\mathbf{O}})$ | 2 |
| 3 | $!\,C(d)$ | 2 | | | |
| 5 | $!\,B(d^{\mathbf{P}})^{\perp}$ | | 1.2 | $you_3 : C(d)$ | 4 |

Opponent wins