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# Finite and Infinite in Greece and China

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Are the notions of finite and infinite used in similar or in different ways in ancient Greek and Chinese thought? It is typical of a certain naïve approach to the problems of comparing Greek and Chinese science to accept the question posed in such brutally simplistic and vague terms, and then to proceed to answer it by developing either the thesis of a basic similarity or the antithesis of a fundamental contrast between the two ancient societies. Indeed this may happen without due consideration being given to how the term "infinite" itself is to be understood, whether in the loose sense (where it stands merely for the immense or the indefinitely large) or in a strictly defined mathematical one (e.g., uncountability, where it is a property of infinite sets that they are neither increased by addition nor decreased by subtraction). Further ambiguities in the Greek terms will be examined in due course.

The aim of this paper is to take this topic as illustrative of the tasks of comparative history of science. As announced in 1993 in the *Newsletter for the History of Chinese Science*, Professor Sivin and I have embarked on an ambitious and wide-ranging exploration of Chinese and Greek science from circa –300 to circa +200—that is, down to the period when Buddhist and Christian influences came to be a major factor in China and Greece respectively. But both of us are very conscious of how superficial most comparative work to date has been.

It cannot be just a matter of recording certain similarities or differences between the ideas produced by these two ancient civilisations, let alone one of chalking up points on either side, as in those studies that seem chiefly preoccupied by the question of who did or thought what first. *Why* the Chinese should have produced the ideas they did, and why again the Greeks *theirs*, have to be addressed and those questions cannot begin to be answered without a sustained analysis of the conditions under which scientific work was conducted.

Why were the problems defined in the ways they were (often very different ways from those we expect if we do not liberate ourselves from the straitjacket of our categories of "mathematics," "physics" and so on)? What expectations were entertained for an adequate solution? Whom were the scientists trying to convince and how did they hope to do so? What was the institutional framework within which they worked? How were they recruited and organised (if they were organised)? What were their motivations or aspirations, what, indeed, their sources of livelihood: how did they make a living, or did they need to?

In our collaborative explorations Professor Sivin has begun a radical reexamination of the Chinese materials, while I have been concentrating—as will be obvious from my essay here—on the Greek side. Yet each of us has found that the juxtaposition of one another's data brings to light problems that risk remaining invisible to anyone working within just a single tradition. New issues come to the fore, especially to do with the relations, in each case, between the science produced and the society which produced it. Our hope is that the agenda we have set ourselves provides a firmer basis not just for doing comparative history of science, but for doing history of science.

My own forays into comparative studies are profoundly indebted to Professor Sivin's inspiration and willingness to collaborate, and this contribution to the articles in his honour can serve as no more than a very meagre token of what I feel I owe to him.

The finite and the infinite are, potentially, a vast topic, and of course there can be no question of attempting a comprehensive analysis here. Rather I shall focus on how to define the problems to be investigated and the most fruitful line of attack to be adopted in their investigation. This is all the more important in that much of what passes as orthodoxy—and not just on the Greek side, but also on the Chinese—is open to challenge and certainly has to be challenged if we are to fulfil the primary task of clarifying the issues.

We may begin with a critical examination of a number of sweeping generalisations that are to be found in the literature, starting with the simplistic theses I mentioned at the outset. I shall develop an argument in three stages. In the first the emphasis will be on the imagined contrasts between Greece and China, and in the second the focus will shift to points of comparison. This will clear the ground for an attempted radical redefinition of the explananda in my final section, where I shall argue that it is not so much this or that concept or use of the finite or infinite that is important: rather the interest lies in the light the ways they are used can throw on the aims, presuppositions and interactions of Greek, and Chinese, thinkers and on the nature of the philosophical exchanges cultivated in each society.

## I

Thus the first stage of the argument might begin with a set of common assumptions about contrasts. Some might claim that it is obvious that there is a massive difference in emphasis between ancient China and Greece in that the latter tolerated, and even cultivated, the infinite in a whole lot of contexts where the Chinese either deliberately excluded it or, more often, never even considered it. There were plenty of ancient Greeks who considered the universe to be spatially infinite and quite a few who believed that there was an infinity of worlds. Both ideas come in different forms. Thus for the atomists, both matter (the atoms) and the void (where matter is not) are infinite. Again for the Stoics, who denied void *within* the cosmos, there is infinite space *outside* it.[1]

Again, infinite worlds were sometimes conceived as succeeding one another temporally, though others held that at any given time there is an unlimited number of worlds. In quite what form infinite worlds were maintained by some of the earlier Presocratic philosophers, such as Anaximander, is disputed.[2] In the –4th century a pupil of Democritus named Metrodorus is said to have claimed that it is as absurd to believe that one ear of corn would be produced in a great plain as to hold that just one world would be, in the infinite void. That just suggests a very large number of worlds. But our source goes on to make the argument that they are indeed infinite in number, since the principles, *aitia*, from which they are formed (that is, atoms and the void) are infinite.[3]

The topic generated a good deal of speculation about what the other worlds may be like. In the –5th century, Anaxagoras, who held that matter is infinitely divisible, believed that the cosmos contained an immense variety of seeds, and

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[1] See, for example, Stobaeus I 161.8ff, Sextus Empiricus, *Against the Professors* X 3, Simplicius, *On Aristotle's "On the Heavens"* 284.28ff, translated in A. A. Long and D. N. Sedley, *The Hellenistic Philosophers*, vol. 1, *Translations of the Principal Sources, With Philosophical Commentary* (Cambridge: Cambridge Univ. Press, 1987), section 49, pp. 294ff. See further below.

[2] The issue is discussed at some length by W.K.C. Guthrie, *A History of Greek Philosophy*, vol. 1, *The Earlier Presocratics and the Pythagoreans* (Cambridge: Cambridge Univ. Press, 1962), pp. 106ff, who sets out the principal ancient Greek and Latin sources. Cf. also the sceptical views expressed by G. S. Kirk, J. E. Raven and M. Schofield, *The Presocratic Philosophers* (Cambridge: Cambridge Univ. Press, 1983), pp. 122ff.

[3] Aetius I 5 4, quoted by Hermann Diels and Walther Kranz, *Die Fragmente der Vorsokratiker* (3 vols., 7th edition, Berlin: Weidmann), vol. 2, section 70 A 6, pp. 231f.

in fragment 4 he describes the formation of humans and other animals in situations other than the one we are familiar with:

> And [we must suppose that] men have been formed and all the other animals that have life; and the men have settled cities and cultivated fields as with us, and sun and moon and the rest as with us; and the earth grows all sorts of produce for them, the most useful of which they gather into their houses and use. This is my account of the separating off, that it must have taken place not only where we live, but elsewhere also.[4]

The contrast, here, between "as with us" and "elsewhere" is ambiguous: Anaxagoras could be thought to refer not to another world, but just to another part of the earth. But Democritus, as reported by Hippolytus, leaves no doubt that he has other worlds in mind:

> There are innumerable worlds of different sizes. In some there is neither sun nor moon, in others they are larger than in ours and others have more than one. These worlds are at irregular distances, more in one direction and less in another, and some are flourishing, others declining. Here they come into being, there they die, and they are destroyed by collision with one another. Some of the worlds have no animal or vegetable life nor any water.[5]

Spatial infinity was sometimes maintained with an argument, versions of which were put forward by both the Stoics and the Epicureans,[6] though the original may go back to pre-Aristotelian Pythagoreans. Archytas, in the –4th century, is reported to have asked: "If I were at the extremity, say at the heaven of the fixed stars, could I stretch out my hand or staff or could I not?" It would be absurd to think you could not do so. But that means that there will be either body or place outside the supposed extremity—an argument that can be repeated for whatever extreme point is postulated.[7]

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[4] Translation from W.K.C. Guthrie, *A History of Greek Philosophy*, vol. 2, *The Presocratic Tradition from Parmenides to Democritus* (Cambridge: Cambridge Univ. Press, 1965), p. 314. Anaxagoras was also famous, or rather notorious, among the Greeks, for having proclaimed that the sun and moon themselves are "incandescent stones": the ancient evidence is set out by Guthrie at pp. 307ff.

[5] Hippolytus, *Refutation of the Heresies* I 13 2, in Diels and Kranz, *Die Fragmente der Vorsokratiker*, vol. 2, section 68 A 40, p. 94. Translation from Guthrie, *A History of Greek Philosophy*, vol. 2, p. 405.

[6] See Simplicius, *On Aristotle's "On the Heavens"* 284.28ff, Lucretius, *On the Nature of Things* I 958ff, translated in Long and Sedley, *The Hellenistic Philosophers*, vol. 1, at pp. 295 and 44f respectively.

[7] See Simplicius, *On Aristotle's "Physics"* 467.26ff, in Diels and Kranz, *Die Fragmente der Vorsokratiker*, vol. 1, pp. 430f, where Simplicius claims to be drawing on the

Temporal infinity, in turn, was even more common, in that some who thought the world spatially finite held nevertheless that it is eternal. Here too different positions were adopted. Some held that the universe has neither beginning nor end, others merely that it has no end. Aristotle took the first view and argued that any supposed beginning to the cosmos must itself have a *prior* beginning, leading to an infinite regress of beginnings and so to the denial of any *first* one. Plato presents the second option, in his cosmological dialogue the *Timaeus*, where it is emphatically stated that the cosmos came to be, though it will not be destroyed. Already in antiquity, however, some thought that that statement that the cosmos came to be was only introduced for the sake of the narrative exposition in the dialogue, and did not represent Plato's own belief.[8]

So far I have taken cosmological examples. But Greek mathematics provides many more examples—so it could be argued—where the infinite is accepted and used readily enough. This could be said to be the case in arithmetic, with the number series, including such famous arguments as Euclid's proof of the infinity of primes (*Elements* IX 20), and in geometry, where the standard Greek view is that geometrical space is infinitely divisible.

Some of these Greek ideas no doubt find parallels in ancient Chinese thought. But the classical Chinese were evidently less inclined to try to prove a strict spatial infinity and less prone to speculate about an infinite number of worlds separated from ours in either space or time or both. So at a first stage it might seem that the contrasts between Chinese and Greek thought on this topic are overwhelming—and to those to whom they do, that no doubt would be the moment when speculative explanations would be invoked to *account* for those differences. But that would be premature.

## II

At a second stage of reflection much of the above picture, both of the Greeks and of the Chinese, has to be modified. Let me deal first with Greek physics, then with Greek mathematics, before turning to some of the Chinese materials.

Both the belief in spatial infinity and that in infinite worlds can, it is true, be found. But two points need to be made even in this regard. First, there is a lexical point that is relevant to the whole of our analysis of Greek thought on the

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–4th century historian Eudemus. Translation from Guthrie, *A History of Greek Philosophy*, vol. 1, p. 336, who discusses other Pythagorean ideas on time and the unlimited.

[8] For Aristotle's own view, see *Physics* VIII chaps. 5 and 6, e.g. 258b10ff, *On the Heavens* I chaps. 10–12. For Plato's statement, see *Timaeus* 28b, and for the ancient dispute over that, see Aristotle, *On the Heavens* 279b32ff and the other texts collected and discussed by A. E. Taylor, *A Commentary on Plato's Timaeus* (Oxford: Clarendon Press, 1928), pp. 67ff.

subject. The Greek word often translated "infinite" is *apeiron*, which combines a negative alpha with the term *peras* meaning limit. But *apeiron* often carried the weaker sense of *indefinite*. So maintaining *apeiroi kosmoi* sometimes implied no more than a belief in a large number of worlds, an indefinite number in fact. However it has also to be said that in other contexts, infinite in the strict sense *is* the right translation of *apeiron*, and not just in some of the mathematical work. Archytas' argument, for instance, suggests that the notion of an extreme point is self-contradictory.

Secondly, it might be argued that those who made most use of the notion of the infinite in cosmological and physical speculation were the atomists, Leucippus and Democritus in the –5th century, Epicurus and his followers in the –4th and later centuries. But atomism, it might further be claimed, was never the *dominant* tradition in Greek philosophy and science, which is represented, rather, by Plato and Aristotle in the mid –4th century, and then by the Stoics, from the late –4th century onwards. Plato, Aristotle, and such Stoics as Zeno of Citium and Chrysippus disagreed on a number of issues, but all were continuum theorists and all teleologists.

The assumption that many of the Greek cosmologists made was that the sphere of the fixed stars marked the limits of the cosmos and this was shared by most of the astronomers. Otherwise why would Archimedes have tried to calculate how many grains of sand it could hold? Of course Archimedes' interest is in developing a mathematical notation to express very large numbers, and it is for that purpose that he conducts his thought experiment with the grains of sand idea. Indeed he sets out by denying both that the grains of sand would be infinite and that they exceed any expressible number.[9] But practising astronomers too, down to Ptolemy in the +2nd century and beyond, assumed that the so-called fixed stars bound the cosmos, and while that did not rule out the Stoic view of infinite space outside the cosmos, for some—as for Aristotle—cosmos and universe are alike bounded and coterminous.

The first-stage presentation of Greek thought on the infinite might be criticised as seriously misleading in that many Greek philosophers, including some of the most prominent ones, fought shy of the notion of the infinite in many of its forms. Aristotle might be cited as a prime case in point.[10] True, he allows, as

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[9] Archimedes, *Sand-Reckoner*, II 216.2ff, translated in T. L. Heath, *The Works of Archimedes Edited in Modern Notation, with the Method of Archimedes* (Cambridge: Cambridge Univ. Press, 1912), pp. 221ff.

[10] This was the view expressed by Melbourne G. Evans, *The Physical Philosophy of Aristotle* (Albuquerque: Univ. of New Mexico Press, 1964). Contrast, for example, Jaakko Hintikka, *Time and Necessity: Studies in Aristotle's Theory of Modality* (Oxford: Clarendon Press, 1973); Wilbur R. Knorr, "Infinity and Continuity: The Interaction of Mathematics and Philosophy in Greek Antiquity," in Norman Kretzmann (ed.), *Infinity and Continuity in Ancient and Medieval Thought* (Ithaca, N.Y.: Cornell Univ. Press,

we have noted, that the universe is temporally infinite. But he opts firmly for a *spatially* limited universe. The heavens form a sphere with the heavenly bodies moving with eternal circular motions in the superlunary region. The earth is at the centre of the entire system, and there can, in this picture, be no question of a plurality of worlds. In *On the Heavens* Aristotle first devoted three chapters to showing that there can be no infinite body, neither simple nor composite,[11] and then proves that there cannot be more than one world.[12] This he does in part on the basis of his analysis of natural motions, which he claims must be either rectilinear—to or from the centre of the universe—or circular. That consideration also provides him with one of his arguments to show that the earth itself is spherical, but in this context he is concerned to establish that there can be only one centre and only one circumference to the world as a whole.[13]

Moreover, so far as the infinite in mathematics goes (to which I shall be returning) Aristotle's view is that the mathematicians do not need the number series *actually* to be infinite: all they need—so he believed, wrongly, as has recently been claimed[14]—is the idea that that series can be indefinitely extended. Even when it comes to the geometrical and physical applications of the notion of the continuum, where Aristotle is clear that it *is* a matter of infinite divisibility, the infinite in question, there, is, in both cases, potential, never actual. You can cut a line or a stick wherever you like. But Aristotle resists the idea (that he nevertheless records) that it is sensible to think of an actual division everywhere of what can potentially be divided anywhere.

There is a famous argument in *On Coming-to-be and Passing Away* that may give an atomist's point of view.[15] Suppose that magnitudes *are* infinitely divisible. Then let them be divided everywhere. What have you left? You cannot be left with *nothing*, nor with what is dimensionless such as a point (since in both cases summing nothings, or what is dimensionless, gives nothing). Nor yet can you say you are left with a magnitude, because *ex hypothesi* you are supposed to have carried out the division *everywhere*. That may have been an argument that led the atomists to the conclusion that magnitudes (of any sort, or at least physical ones) must be constituted from *indivisible* atomic quanta.[16] But Aristotle simply denies that it could ever be the case that a magnitude is divided

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1982), pp. 112–45; Richard Sorabji, *Matter, Space and Motion* (London: Duckworth, 1988).

[11] Aristotle, *On the Heavens* I chaps. 5–7, 271b1ff.

[12] Aristotle, *On the Heavens* I chaps. 8–9, 276a18ff.

[13] Aristotle, *On the Heavens* 276b7-21, cf. II chap. 14, 297a8ff, on the sphericity of the earth.

[14] Hintikka, *Time and Necessity*, pp. 118ff, on Aristotle, *Physics* 207b27ff.

[15] Aristotle, *On Coming-to-be and Passing Away* 316a14ff.

[16] The answer to the question of how far Leucippus and Democritus had been anticipated by earlier views postulating atomic quanta depends on the interpretation of the evidence for Zeno and for the Pythagoreans: see further below, Note 44.

"everywhere." That is just something potential, never actual, even though there is some difficulty in squaring that view with Aristotle's insistence elsewhere that what is potential must be thought of as actualisable and that must mean at some point actualised—the idea that has been dubbed the principle of plenitude.[17]

A similar potentially tricky deployment of the distinction between potential and actual infinities appears also in Aristotle's critique of Zeno. In the so-called stadium argument,[18] reported by Aristotle at *Physics* 233a21ff and 239b9ff, Zeno had claimed (1) that to cross a stadium a runner must touch infinitely many points in the ordered sequence 1/2, 1/4, 1/8 and so on, traversing on each occasion half of the total space remaining to be crossed; (2) that it is impossible to touch infinitely many points in a finite time; therefore (3) the runner cannot reach his goal. But Aristotle replies that the finite time itself, like the space to be traversed, is infinitely divisible: so (2) can be denied, and the conclusion (3) also. But while he believes that that is a sufficient argument *ad hominem*, to answer a questioner who raises the point at issue in (2), he comes back to the problem in a later passage, *Physics* 263a11ff, that reveals his continuing worries over the distinction between potential and actual infinities:

> So when someone asks the question whether it is possible to traverse infinite things, either in time or in distance, we must reply that in a way it is but in a way it is not. For if they exist actually, it is not possible, but if potentially, it is; for someone in continuous movement has traversed infinite things incidentally, not without qualification; for it is incidental to the line to be infinitely many halves, but its essence and being are different.[19]

Aristotle's difficulty is as before: the continua of space, time and motion are all infinitely divisible, for that is what it is to be a continuum. Yet they can be infinitely divisible only in potentiality. That move gives Aristotle his reply to those arguments of Zeno, and yet that leaves the relationship between the potential and the actual, in these cases, unclear, given Aristotle's usual insistence that the potential corresponds to what can indeed be actualised.

But as a source of qualifications to the contrasts suggested between China and Greece in my first-stage argument, the Stoics are just as important as Aristotle. Outside the cosmos, as noted, they hold that there is infinite void. But so far as the cosmos itself goes, they too hold that there is just the one, even though

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[17] Arthur O. Lovejoy, *The Great Chain of Being: A Study of the History of an Idea* (Cambridge, Mass.: Harvard Univ. Press, 1936); cf. Hintikka, *Time and Necessity*, chap. 5.

[18] This analysis is based on that of Kirk, Raven and Schofield, *The Presocratic Philosophers*, pp. 269ff. The so-called Achilles argument, reported by Aristotle at *Physics* 239bff, similarly depends on the assumption that space is infinitely divisible.

[19] *Physics* 263b3ff, translation from Kirk, Raven and Schofield, *The Presocratic Philosophers*, p. 271.

it is subject to periodic conflagration.[20] Furthermore they are continuum theorists in a strong sense. The cosmos is finite and there is no void within it. Unities come in different forms (the unity represented by a living organism is greater than that of inanimate substances) but everything forms part of a single interconnected whole. That different parts of that whole resonate with one another is the basis of their doctrine of *sumpatheia*, sympathy, which is made to do some explanatory work in a number of different contexts, in biology, in medicine and in physics.[21] More remarkably still, from the point of view of those looking for parallels between Greece and China, the Stoics held that *pneuma* permeates the whole cosmos: as such it could be thought to be the nearest the Greeks got to 氣 *qi*.

The very fact that Stoic *pneuma* is so badly *mis*represented by their opponents (Plutarch, Galen, Alexander of Aphrodisias, all the way down to Simplicius and Philoponus in the +6th century) is interesting and indicative. The questions that the opponents keep pressing include how many elements there are, what the relation between fire and the rest is, how breath can pervade everything, and what the relation is between its different modalities or manifestations, in inanimate substances, in plants and in animals ("tenor," *phusis, psyche*). But it is clear that the Stoics worked rather with a notion of active and passive principles, not with one of four static elements. At that point a general similarity with *yin* and *yang* might suggest itself, even though the Stoics never developed the system of correspondences with which those principles eventually came to be associated in China.[22] Long and Sedley remark that, for the Stoics, "four distinct elements . . . are not, as they are in Aristotle's cosmology, permanent features of the Stoic universe, but the basic qualifications of matter throughout the duration of each temporally limited world-order."[23] That is correct in general terms, although one may question just how far those elements are indeed "distinct." Even so, the more important difference from Chinese *yin-yang* is that

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[20] See, for example, Nemesius 309.5ff and other evidence collected by Long and Sedley, *The Hellenistic Philosophers*, vol. 1, section 46, pp. 274ff, and section 52, pp. 308ff.

[21] See, for example, Alexander, *On Mixture* 223.25ff and other texts discussed by Long and Sedley, *The Hellenistic Philosophers*, vol. 1, section 47, pp. 280ff.

[22] See, for example, Joseph Needham, *Science and Civilisation in China*, vol. 2, *History of Scientific Thought* (Cambridge: Cambridge Univ. Press, 1956), pp. 253ff; John B. Henderson, *The Development and Decline of Chinese Cosmology* (New York: Columbia Univ. Press, 1984), chap. 1, pp. 1–58; Angus C. Graham, *Yin Yang and the Nature of Correlative Thinking* (Singapore: Institute of East Asian Philosophies, 1986); Angus C. Graham, *Disputers of the Tao: Philosophical Argument in Ancient China* (La Salle, Ill.: Open Court, 1989), pp. 319ff.

[23] Long and Sedley, *The Hellenistic Philosophers*, vol. 1, p. 286. The Stoic evidence for elements and unities is collected at pp. 280ff.

they, *yin-yang*, are essentially functional and relational, and as such *not* basic qualifications of matter.[24]

Then there is a further feature of Greek philosophy that can be taken to suggest a resistance to the notion of infinity at least in certain contexts. This is the recurrent idea, found for example in Plato, Aristotle and Plotinus, that the infinite is not *intelligible*: whatever has to be called *apeiron* is, as such, no proper subject of inquiry and understanding. There are texts in Aristotle that say as much in so many words. Thus in the *Physics* 187b8f he puts it that "the infinite qua infinite is unknowable: what is infinite in multitude or size is unknowable in quantity, and what is infinite in form is unknowable in quality." He uses a similar argument in the *Metaphysics* 1036a8ff, 1037a27ff, to suggest that matter, as indeterminate, is unknowable in itself, and again at *Physics* 207a14ff, 25ff, he attacks Melissus for having asserted that the whole is infinite, repeating the point that the infinite as such is unknowable.

Plato too equates what can be known with what is limited or determinate. In a late and rather neglected dialogue, the *Philebus*, that draws on Pythagorean ideas, especially in the fundamental contrast it uses between limit and the unlimited, Plato makes it clear that the world as a whole, and its parts, can be understood only insofar as it, or they, manifest limit. True, there is also what is there to be limited, the as-yet-to-be-determined, that of which the limit is the limit. But it is the determinate or what has been determined that is there to be known.[25]

Finally the +3rd century neo-Pythagorean Plotinus was to go so far as to make explicit what had been implicit already in the Pythagorean Table of Opposites reported by Aristotle in the *Metaphysics* 986a22ff, namely that the infinite is evil, *kakon* (*Enneads* VI 6 1).

But it is not just Greek philosophy that provides evidence to suggest that certain important qualifications need to be entered before we accept the idea that Greek thinkers tolerated and used the notion of infinity with equanimity. Reservations also need to be expressed with regard to our earlier, first-stage, characterisation of Greek mathematics.

First it is well known that what is called, most misleadingly, the "method of exhaustion" *avoids* infinite processes. The method, usually thought to have been developed by Eudoxus in the –4th century, depends on the principle expressed in Euclid *Elements* X 1: "Two unequal magnitudes being set out, if from the greater there be subtracted a magnitude greater than its half, and from that which is left a magnitude greater than its half, and if this process be repeated continually, there will be left some magnitude which will be less than the lesser

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[24] See Nathan Sivin, "Yin Yang and the Five Phases," in G.E.R. Lloyd and Nathan Sivin, *Tao and Logos: Comparing Ancient Chinese and Greek Science* (forthcoming).

[25] Plato, *Philebus* 16c–17a, 25a.

magnitude set out."[26] Used, as for example in *Elements* XII 2, in the investigation of the ratios of circles to their diameters, it allows the inscription of regular polygons in a circle such that their area can be made to approximate *as close as one likes* to the area of the circle. But the area is, precisely, *never* exhausted. This gives a proof procedure that is rigorous, but involves no breach in the continuity axiom. There is no suggestion that the circle can be identified with the infinite-sided regular polygon inscribed within it. The Greek preference for the method of exhaustion is thus evidence *both* of their demand for rigour *and* of their avoidance of infinite processes wherever possible.

When, as in Archimedes' *Method*, the continuity axiom is breached at least insofar as a curvilinear area is there treated as made up of the indivisible line segments it contains, this is recognised as disqualifying the procedure as demonstrative: it can be no more than heuristic. Archimedes is clearly on the defensive, not just because of the use of indivisibles, but also because he treats plane figures as having determinate centres of gravity and as balanced around a fulcrum. Judged by the usual standards of Greek demonstrative rigour, the method falls short. But it is equally remarkable that, if not demonstrative, it is at least accorded heuristic status, for it was, after all, in conflict with the common Greek assumption of the infinite divisibility of geometrical magnitudes.

Archimedes also provides us with most of our not very extensive evidence for the summing of infinite series in Greek mathematics. The lack of a clear distinction between converging and diverging series had caused difficulty already in the days of Zeno of Elea.[27] Nor perhaps should we be surprised that the examples of Greek mathematicians confidently summing infinite series are rare,[28] when we reflect that, expressed in Greek, the notion of taking the limit

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[26] Translation from T. E. Heath, *The Thirteen Books of Euclid's Elements* (3 vols., 2nd edition, Cambridge: Cambridge Univ. Press, 1926), vol. 3, p. 14. See Wilbur R. Knorr, *The Evolution of the Euclidean Elements* (Dordrecht: D. Reidel, 1975), pp. 256f, 271ff, who argues that the stipulation "more than half" may suggest an origin in relation to the anthyphairetic approach to incommensurables; cf. David H. Fowler, *The Mathematics of Plato's Academy: A New Reconstruction* (Oxford: Clarendon Press, 1987).

[27] Thus in the arguments (Fragments 1 and 2) attacking the many on the grounds that they must be "both so small as to have no size and so large as to be unlimited" (Simplicius, *On Aristotle's "Physics"* 141.8), there is an ambiguity in the term "unlimited." Most ancient commentators took it to mean "indefinitely large." But that had not been shown. Zeno had argued that there is an infinite number of parts: but the sum of a—converging—infinite series may be finite. It is controversial whether Zeno himself is under an illusion on the point, or whether it is rather just his opponents who may be. But either way the conclusion could easily have been resisted by an appeal to the distinction between converging and diverging series.

[28] Thus Knorr, "Infinity and Continuity," p. 125 n. 30, cited Archimedes, *On the Quadrature of the Parabola*, Proposition 23, as unique: but that is to ignore other cases; cf. Tohru Sato, "A Reconstruction of The Method Proposition 17, and the Development

(*peras*) of any series that is limitless (*apeiron*) certainly ran the risk of appearing flatly self-contradictory.

So much for some of the reservations that should be expressed with regard to our initial characterisation of Greek thought. But equally on the Chinese side, any thesis that would have it that Chinese thought in general was reluctant, even incapable, of dealing with the infinite, encounters considerable counter-examples. It is true that the evidence for some uses of infinity in Chinese thought is late or unreliable or both. That applies especially to cosmology, where many invocations of the notion of the infinite, such as, for instance, the idea of infinite worlds, betray clear Buddhist influence,[29] and so fall outside the period with which I am chiefly concerned. Again for the so-called 宣夜*xuanye*, cosmology, we have a mid +4th century source, Ge Hong, who reports the view that the heavens are empty, immensely high and far away, and without bounds. But the writer does not inspire confidence in that he states that the books that set out this view were all lost, even though he claims to be reporting what a late Han librarian, Qi Meng, said its earlier masters had taught.[30]

However, in other contexts we are on firmer ground. In a variety of areas of Chinese thought we have good evidence that recurrent procedures of different types were used with some confidence. One such text comes in the paradoxes reported towards the end of the *Tianxia* chapter (33) of *Zhuangzi*.[31] They begin with a set of ten directly attributed to Hui Shi himself, and several of these evidently challenge conventional notions of space and time. Thus the first states: "The ultimately great has nothing outside it. . . . The ultimately small has nothing inside it," and the second: "The dimensionless cannot be accumulated, yet its girth is a thousand *li*."[32] The contexts of these remarks are not recorded either here or in our other ancient evidence for Hui Shi, and modern scholarly interpretations diverge radically.[33] But Hui Shi's own paradoxes are then followed by others, including: "if a stick a foot long is cut in half every day, it will not be

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of Archimedes' Thought on Quadrature, Part II," *Historia Scientiarum* no. 32 (1987), pp. 75–142 at pp. 114ff.

[29] See, for example, Jacques Gernet, "Space and Time: Science and Religion in the Encounter Between China and Europe," *Chinese Science* no. 11 (1994).

[30] Joseph Needham, *Science and Civilisation in China*, vol. 3, *Mathematics and the Sciences of the Heavens and the Earth* (Cambridge: Cambridge Univ. Press, 1959), p. 219, cites Ge Hong from the *Taiping yulan* 2, 7a, and *Jinshu* chap. 11, 2a. I am grateful to Nathan Sivin for stressing to me the difficulties of accepting this report.

[31] *Zhuangzi* 33: 70ff.

[32] Translation from Graham, *Disputers of the Tao*, p. 78.

[33] With Needham, *Science and Civilisation*, vol. 2, pp. 190–91, compare Jean-Paul Reding, *Les Fondements philosophiques de la rhétorique chez les sophistes grecs et chez les sophistes chinois* (Bern: Peter Lang, 1985), pp. 350ff, 435ff, and Graham, *Disputers of the Tao*, pp. 78ff.

exhausted in ten thousand generations."[34] Once again we are at a loss for the precise context, but what is beyond doubt, in this instance, is that we have a clear appeal to a recursive bisection principle, similar in that respect at least to those attested for Zeno of Elea.

Our evidence from the Chinese mathematical treatises is both richer and more complete, even though, to be sure, not without its problems of interpretation. Thus in Liu Hui's commentaries on the *Nine Chapters* recurrent procedures are used both in the investigation of the area of the circle in chapter 1 and in that of the volume of the pyramid in chapter 5.[35] In chapter 1 he first establishes that the area of the circle is given by the formula, $A$ = 1/2 $c$ times 1/2 $d$, where $A$ is the area, $c$ the circumference and $d$ the diameter. He then proceeds to an approximation of the ratio between the circumference and the diameter (that is p). In this a hexagon is first inscribed in the circle, then a dodecagon, and the procedure of doubling the sides of the inscribed polygon is then continued to give increasingly accurate approximations to the area of the circle itself. In chapter 5, similarly, Liu Hui proceeds by inscribing in the pyramid to be determined, figures that approximate closer and closer to it (see Appendix).

In chapter 1 he remarks: "the finer it cuts, the smaller the loss [or error, 失 *shi*]; if one cuts it and further cuts, until one reaches what one can no longer cut, then it coincides [fits] with (合體 *he ti*) the circle and there is no error."[36] Here Liu Hui *may* envisage the convergence of the perimeter of the inscribed polygon and the circumference of the circle, though it is clear from the subsequent discussion that it is recognised that with any determinate-sided polygon there is a remainder. Precisely which sections of the ensuing investigation of the value of the circumference-diameter ratio are Liu Hui's own work and which the work of later commentators is disputed,[37] but their efforts are directed at better and better approximations and never claim an *exact* value.

Moreover in the otherwise similar investigation in chapter 5, Liu Hui uses a rather different formula for his result. "The smaller they are halved, the finer are the remaining [dimensions]. The extreme of fineness is called 'subtle' (微 *wei*); that which is 'subtle' is without form (形 *xing*). When it is explained in this way, how could one get a remainder?"[38]

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[34] *Zhuangzi* 33: 78.

[35] In tackling the problems addressed in this section I have benefited a great deal from an extensive exchange of correspondence with Dr. Donald Wagner.

[36] Chap. 1: 103–104. Translation adapted from Li and Du, *Chinese Mathematics*, p. 68. Cf. Chemla, "Méthodes Infinitésimales," p. 41 n. 15.

[37] See especially Donald B. Wagner, "Doubts Concerning the Attribution of Liu Hui's Commentary on the *Chiu-Chang Suan-Shu*," *Acta Orientalia* no. 39 (1978), pp. 199–212, at pp. 206ff.

[38] Chap. 5: 168. Translation adapted from Wagner, "An Early Chinese Derivation," p. 173. Cf. Karine Chemla, "Méthodes Infinitésimales en Chine et en Grèce Anciennes: les limites d'un parallèle," in J. M. Salanskis and H. Sinaceur (eds.), *Le Labyrinthe du*

Both the similarities with, and the differences from, Greek procedures are alike remarkable. In neither case does Liu Hui proceed, as the Greeks would normally have done, via an indirect proof.[39] The Greeks would have demonstrated that, for example, the area to be determined cannot be either greater or less than a given area and so must be equal to it.[40] Liu Hui's attack is direct, via the securing of closer and closer approximations.

Again the Greeks, as we have seen, generally thought that the area to be determined is, precisely, *not* exhausted, for Eudoxus' principle allows the insertion of a further rectilinear figure between any rectilinear figure and the circumference to which it approximates. True, there is one notable Greek exception, namely Antiphon, who is reported to have claimed that the circle is "at some point" (*pote*) exhausted by the inscribed polygons.[41] But his suggestion was rejected by philosophers and mathematicians alike as being in conflict with the principles of geometry, that is to say the continuum assumption. Aristotle curtly notes (*Physics* 185a16f) that whereas other attempts to square the circle need refuting, "it is not the business of the geometer to refute that of Antiphon."[42]

Liu Hui, for his part, is clear, as we noted, that with any polygon with a determinate number of sides, its perimeter falls short of the circumference of the circle. Both here and in the pyramid investigation he envisages iterations that can be continued indefinitely. It is interesting, however, that there is a certain variation or hesitation in the strength of the claims made at the end of the inquiry. In chapter 1 you continue until no further cuts are possible and "there is no error," while in chapter 5 the conclusion is a question: "how could one get a remainder?" Moreover the mathematical results obtained in the two cases differ. In chapter 5 the formula for the volume of the pyramid is *exact*. In the circle division, while the formula $A = 1/2\ c$ times $1/2\ d$ is also exact, that is no use until a value for the length of the circumference is obtained: in fact the numerical value for the circumference-diameter ratio is no more than approximate, though it can be made increasingly accurate.

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*continu* (Berlin: Springer, 1992), pp. 38f, who translates the final phrase: "comment obtiendrait-on un reste?"

[39] This point has been made by Chemla, "Méthodes Infinitésimales," pp. 42ff.

[40] As in Euclid's investigation of the circle in *Elements* XII 2 (cf. also Archimedes, *On the Measurement of the Circle*) and the discussion of the pyramid at *Elements* XII 3.

[41] As reported in Simplicius, *On Aristotle's "Physics"* 54.20ff (note *pote dapanomenou* at 55.6), on which see Ian Mueller, "Aristotle and the Quadrature of the Circle," in Norman Kretzmann (ed.), *Infinity and Continuity in Ancient and Medieval Thought* (Ithaca, N.Y.: Cornell Univ. Press, 1982), pp. 146–64 at pp. 154ff; Knorr, "Infinity and Continuity," pp. 130ff.

[42] The contrast is with Hippocrates of Chios, at least according to the ancient commentators, Themistius 3.30ff; cf. Simplicius, *On Aristotle's "Physics"* 54.12ff, though it is doubtful if Hippocrates was guilty of any fallacy; see G.E.R. Lloyd, "The Alleged Fallacy of Hippocrates of Chios," *Apeiron* no. 20 (1987), pp. 103–28.

The question then arises as to whether Liu Hui's expressions imply that he has a clear grasp of the notion of taking the limit of an infinite series, and we have to stress that much of the background we need to answer that is lacking. We simply do not have good contemporary texts that could throw light on the extent to which a technical vocabulary had been developed in contemporary mathematics. The key question is the expression "there is no error," which can be taken either loosely—there is no appreciable error, the error can be discounted—or strictly—there is no error since what is envisaged is the strict coincidence of the infinite-sided polygon with the circle. On the one hand there is no talk of infinite sides and no clear Greek-style definition of limit. On the other the term *wei* is clarified in chapter 5 and not just for use in that investigation since it is applied elsewhere in the commentary. What Liu Hui invokes in the pyramid determination is an "extreme of fineness" that is "without form." That expression too can be taken strictly or loosely, but offered as an *explanation* for why there is no remainder may suggest Liu Hui's confidence that the formula of the volume of the pyramid (that is, one third length times breadth times height) is exact—as indeed it is. If the variation in the expressions used in the two cases is *mere* variation, for stylistic purposes, that would be compatible with the hypothesis that Liu Hui is in complete control, on both occasions, of procedures that imply the taking of a limit to an infinite series.

In the absence of more determinate evidence we have to suspend judgement on that point. Nevertheless, for our purposes here, the upshot of this rapid survey is clear. It is obvious that Liu Hui is as much at home as any Greek geometer with the deployment of indefinitely recurring procedures, and it is possible that he was a good deal more at home than most with the notion of convergence to a limit.

## III

So at the second stage in our argument we can say that good grounds have been identified for revising quite drastically a number of the generalisations about Greece and China propounded at the first stage in the discussion. However, a more radical approach to the issues needs to be adopted. What both lines of argument so far developed have in common, and where *both* seem flawed is in this: they focus on similarities and differences in *concepts* or *theories* (between Chinese and Greeks) and then strive to identify, on the Chinese or on the Greek side, what the majority of thinkers, or the most important ones, maintained in most contexts, and they then attempt to use what has been identified as *the* Chinese or Greek view as the basis for an argument insisting either on the contrasts (as in stage one) or on the similarities (stage two). But—quite apart from the risky, even delusory, character of any generalisation to what *the* Chinese or *the* Greek view is—that may miss the point.

On the Greek side, first, it is easy to see that on this subject, as on so many others, there was no uniformity, let alone orthodoxy. Almost any position, any use of infinity, can be attested, and most theses that were put forward by ancient Greek thinkers were challenged by (other) Greek thinkers themselves. So it is no wonder that both the claim that the Greeks espoused the infinite with equanimity, and the claim that they fought shy of it, have some evidence that can be cited in apparent support. But that does not get us anywhere.

But more importantly, secondly, the Greeks often maintained whatever position they took up on these issues in direct opposition to the alternative. First, they were generally conscious of the alternative, but second, and again more importantly, they often generate arguments for their own positions *by way of* refuting those alternatives.

It is not as if Plato and Aristotle simply ignored the opposition: they *used* it. Aristotle, especially, develops atomistic arguments then to put them down—as we have seen in the passage from *On Coming-to-be and Passing Away* cited before. The same dialectical confrontations recur in the Hellenistic period, though the quality of the polemic varies. Thus Epicurus sometimes proceeds by simply denying the opposition's case or dismissing it out of hand, though there are also plenty of arguments against rival views in him and in one of our fullest later sources for Epicureanism, the +1st century Roman poet Lucretius. Again I have remarked that neither Galen nor Alexander is exactly fair to the Stoic view of matter when they attack it. However, the point remains that there continues to be a heavy use of whatever opposition you face, not just for destructive purposes, to show that *they* are mistaken, but also for constructive ones, to build up *your own* position. The alternatives, indeed, were often seen as mutually exclusive and exhaustive: that allowed arguments to proceed by exclusion. It cannot be the case that the universe is finite/infinite: *so* it must be infinite/finite. It cannot be the case that matter, space and time are infinitely divisible: so they must be constituted by indivisibles. Or vice versa.[43]

The dialectical confrontations get an added twist with the Hellenistic Sceptics. Dilemmatic arguments presenting antinomies were used already, long before, notably by Zeno of Elea, who aimed to defeat the assumption that motion is possible *whatever* view was taken on how space and time were composed.[44]

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[43] I explored the use of this and other types of "polar" arguments in G.E.R. Lloyd, *Polarity and Analogy: Two Types of Argumentation in Early Greek Thought* (Cambridge: Cambridge Univ. Press, 1966).

[44] This remains the case, even though Zeno's presentation of the dilemma was not as crisp as Paul Tannery, *Pour l'histoire de la science hellène* (Paris: Gauthier-Villars, 1930) thought when he envisaged Zeno attacking motion first on the assumption that space and time are infinitely divisible, then on the assumption that they consist in strict indivisibles. The second limb of the reconstruction faces two difficulties: (1) that it conflicts with how the ancient commentators took Zeno, (2) the notion of strict quanta of time is otherwise not attested before the late –4th century. But even if Zeno did not use

But the Hellenistic Sceptics held that on every theoretical issue, relating to underlying reality or hidden causes, *both* sides were wrong.[45] One should, as they said, suspend judgement. But the way they proceeded was, precisely, to match every positive ("dogmatic") thesis to its antithesis—to leave not just one of them, but both, undermined. *Isostheneia*, equal strength on either side, is what they called this: but if equally strong, equally weak.

Here then, in stage three of our discussion, we rejoin some familiar topics on the Greek side, at least, namely the antagonistic strands that are such a feature of Greek thought, together with the recurrent concern for foundational questions and the ultimate justification for a position.[46] In the latter regard, both atomic and continuum theories addressed the issue of the *ultimate* constitution of matter, space and time. It is striking that continuum theorists tend to be continuum theorists *across the board*—and so too were atomists.[47] That is, when a continuum view was adopted, it was adopted not just for physical matter, but also for space and time and indeed for geometrical magnitudes: all were treated as infinitely divisible. Similarly the atomists often deny all these modalities of continua, and postulate atomic quanta, physical, spatial, temporal, geometrical, instead. Mixtures of the two types of position are rare, if not (as Furley suggested) quite unknown: one possible exception is Democritus, who *may* have combined physical atomism with an assumption of a geometrical continuum, though the point is certainly controversial.[48]

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the idea of strict quanta of time, the arguments called the Arrow and the Moving Rows attack motion on the basis of some looser notion of the component elements of space and time. See, for example, David J. Furley, *Two Studies in the Greek Atomists* (Princeton: Princeton Univ. Press, 1967), pp. 71ff; G.E.L. Owen, "Zeno and the Mathematicians," in *Logic, Science and Dialectic* (London: Duckworth, 1986), pp. 45–61 (originally in *Proceedings of the Aristotelian Society* no. 58 [1957–8], pp. 199–222). The further suggestion, developed by Tannery and others, that Zeno's target was a specific Pythagorean doctrine according to which numbers, points and atoms are identified, is nowadays discounted for lack of reliable primary evidence.

[45] Some of the primary texts are collected in Long and Sedley, *The Hellenistic Philosophers*, vol. 1, sections 68–72, pp. 438ff, and cf. the essays collected in Malcolm Schofield, Myles Burnyeat and Jonathan Barnes (eds.), *Doubt and Dogmatism* (Oxford: Clarendon Press, 1980), and in Myles Burnyeat (ed.), *The Skeptical Tradition* (Berkeley: Univ. of California Press, 1983).

[46] I have discussed aspects of this in G.E.R. Lloyd, *Demystifying Mentalities* (Cambridge: Cambridge Univ. Press, 1990).

[47] As has recently been argued by David J. Furley, *The Greek Cosmologists*, vol. 1 (Cambridge: Cambridge Univ. Press, 1987).

[48] The ways in which these positions may have been reconciled have been discussed by Kate Meakin, "Pre-Platonic Ontology of Mathematics," Ph.D. dissertation, Cambridge, 1990.

I am tempted to argue, therefore, that the plurality of theoretical positions different Greeks adopted on this set of issues is the—all too predictable—outcome of their fondness for the dialectical exploration of abstract ideas. That in turn relates (1) to the rivalry between those who claimed special knowledge and (2) to their sense of how to present their cases in the best possible light in relation to the audiences they were hoping to persuade—often general audiences, indeed, present at open debate between rival views set out by contending parties. *Advocacy*, practised extensively in the Greek law-courts, comes to be a prominent feature of many Greek intellectual exchanges, when opposing "masters of truth" set out to challenge conventional wisdom and each other's replacements for it.

Standard, conventional or "common-sense" opinions thus get to be overturned and repudiated, in ancient Greece, across the board, and on some unlikely issues, by people who claimed to know better. The sensible assumptions that the earth is flat, the heavens a dome above it, physical objects solid, are confronted with all sorts of rival theories proposed by speculative thinkers of varying degrees of fame and respectability. As for the fine structure of time and space, common assumptions did not really tell people quite what to think. You certainly did not need to have a view on that to order your life according to the cycle of the seasons and the passing of the years. But once Greek speculative thought began in earnest with the Presocratic philosophers, then just about every thesis—and its contradictory—are examined, and quite a few are seriously entertained. On that line of argument, it is not at all surprising that, once Greek philosophy got going, then sooner or later infinite worlds, atomic quanta of time, even indivisible lines, would find proposers and seconders—as well as opponents—in the to and fro of abstract debate.

This argument, on the Greek side, does not, to be sure, resolve the problems, so much as redefine them, setting a new agenda, though now one that is investigable in terms of the types of examination of the social framework of science that Sivin and I aim to undertake. The explanandum then becomes not why this or that idea of the infinite was promulgated in this or that context, but rather what the social and intellectual conditions were that allowed such abstract debate to develop.

But what about China? Does not the complexity of the data we uncovered suggest that here too an analogous point applies? If so, our task should not be to attempt to explain some supposed Chinese readiness, or reluctance, to espouse, or to shun, the infinite as such, but rather to see what light our study of the actual, complex, diverging views on the infinite can throw on the aims and presuppositions of the thinkers concerned, whether in mathematics or in philosophy.

Two tentative suggestions, in that regard, may be followed up briefly in conclusion, the first to do with the balance between theoretical interests and practical applications in Chinese mathematics, the second to do with the conduct of philosophical exchanges in China more generally.

First, it has often been claimed that Chinese mathematics is essentially pragmatic, if not indeed directly practical, in aim, and this has been a standard way of contrasting Chinese mathematics with the dominant Greek tradition represented by Euclid or Archimedes. However, our analysis of the use of recurrent procedures suggests that qualifications need to be recognised. First, to take one example of many, the exploration of the ratio of circumference to diameter is certainly carried far beyond any possible practical application. For practical purposes a value of 3 or 3 1/7 is perfectly adequate. But the commentaries on chapter 1 of the *Nine Chapters* pursue the analysis to the point where the area of a polygon of 192 sides is being determined—and even beyond.[49] Yet we must also acknowledge that the aim of that exploration is not a proof, but the best approximation possible.

Yet that is not to say that nothing like proofs are ever attempted in Chinese mathematics. Liu Hui's preliminary study of the area of the circle *shows* that the formula he states is correct, namely that the area is equal to half the circumference times half the diameter. True, the form of the proof differs from the style of axiomatic-deductive demonstration favoured by Greek mathematics in the Euclidean tradition. However, the fact that axiomatic-deductive demonstration does not figure in Liu Hui nor in any other classical Chinese mathematical text should not blind us to the further point, that what we have in Liu Hui, in this context as in others,[50] are perfectly proper procedures to verify correct results proving them, indeed, in just *that* sense.

Moreover on the question of pragmatics, Liu Hui himself notes, at the end of his pyramid investigation, that "the object [called] *bienao* has no practical use; the shape [called] *yangma* can be long or short, or broad or narrow. Nevertheless without the *bienao* there is no way to investigate the number [that is the volume] of a *yangma*; and without the *yangma* there is no way to know [the other shapes connected with the cone and the truncated cone]. These are primary in application."[51]

Two points are striking about this remark. First, Liu Hui clearly recognises that his study goes beyond what is of direct practical use. But then, secondly, he registers a certain defensiveness on the point. It is as if he feels some obligation to *justify* himself here, as if his assumption, or that of other students, is that

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[49] Chap. 1: 104–6. As noted before, the problem of which parts of this discussion are the work of Liu Hui, which that of later commentators, is disputed.

[50] On different styles of mathematical proof, see Wagner, "An Early Chinese Derivation," pp. 164ff, and Karine Chemla, "Resonances entre démonstration et procédure," *Extrême-Orient Extrême-Occident* no. 14 (1992), pp. 91–129.

[51] Chap. 5: 168. Translation adapted from Wagner, "An Early Chinese Derivation," p. 182.

mathematical investigations *should* prove themselves to be, directly or indirectly, of practical use. In that latter regard, the contrast can still be remarked between Liu Hui's views and those of those Greeks who rated mathematics the higher the more abstract, the further removed from practice, it was.[52]

My second, final, more general point relates to debate. I have stressed elsewhere the antagonistic characteristics of much Greek speculative thought, and they are certainly in evidence in discussions of infinity. Face to face confrontation, often before a general audience, the public who in some cases adjudicated who had won the contest,[53] is, at once, typically Greek and profoundly un-Chinese.

But that is not to say that there is no criticism, sometimes indeed hard-hitting criticism, of rivals in Chinese philosophy and science. Two examples relating to our topic can be used to illustrate this. We noted before that the evidence for the *xuanye* idea of a boundless heaven is late. In Han times, two main cosmological ideas were current, in both of which the world is thought of as finite. In the *Gaitian* system, described in the *Zhou bi suanjing* of the –1st century, the heaven is a circular canopy set over a central earth. In the *Huntian* view the heavens are a complete sphere, with half of this invisible below the earth at any time. Each view was associated with—indeed closely tied to—observational techniques, and they are better considered not so much as cosmological theories, as rather ways of doing astronomy.[54]

Although the two are sometimes combined, the period of the rise of the *Huntian* view is characterised by texts that criticise *Gaitian.* Thus the +10th century encyclopaedia *Taiping yulan* preserves an account of a discussion from the lost work *Xin Lun* by Huan Tan (– 40 to +30). In this the *Gaitian* view is duly set out and then criticised from a *Huntian* perspective, on the grounds, for example, that it could not account for the equality of day and night at the spring and autumn equinoxes, and with a direct appeal to the actual experience of the movements of the sun's shadow in the White Tiger Hall where the purported

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[52] This is a prominent and recurrent theme among those who considered themselves Platonists, such as Proclus, in his commentaries on the *Timaeus* and on Euclid's *Elements* I. But they could and did cite texts from Plato himself as their ancient authority, for example Socrates' remarks on the so-called propaedeutic studies in the *Republic* (e.g. 525aff). Thus it is claimed at *Republic* 530bc that the chief usefulness of astronomical study lies, precisely, in its value in training the soul in abstract thought.

[53] This is the case, for example, with what are called sophistic *epideixeis*, or display speeches, examples of which are extant in our Hippocratic Corpus and which are referred to in *On the Nature of Man,* chap. 1.

[54] For a recent discussion of these two, see Christopher Cullen, *Astronomy and Mathematics in Ancient China: the Zhou Bi Suan Jing* (Cambridge: Cambridge Univ. Press, forthcoming).

discussion took place.[55] While that is still a far cry from Greek debates aiming to establish by argument that the heavens *must* be finite, or again infinite, it illustrates an openness to alternatives and a recognition that they are just that, potential competitors.

As a second example we may revert to the way in which the topic of the infinite provided resources for paradox in China, as it also did in Greece. We mentioned the dicta attributed to Hui Shi and others reported in the *Tianxia* chapter of *Zhuangzi*. However the reaction that Hui Shi provoked, in that discussion, is interestingly different from the reception that later Greek philosophers accorded to Zeno. Aristotle, at least, as we saw, wanted to defeat Zeno by argument, the effect of which was to claim that Zeno had ignored a distinction that he, Aristotle, maintained to be important.

The reaction to Hui Shi in *Zhuangzi* is milder, if condescending, and it proceeds not by an attempted refutation but by lamenting the waste of his talents. "He had many formulas, his writings filled five carts, but his way was eccentric. . . ." "Hui Shi day after day used his wits in disputation with others, but it was only in the company of the disputers in the world that he distinguished himself as extraordinary. . . . What a pity that Hui Shi's talents were wasted and never came to anything, that he would not turn his back from chasing the myriad things. He had as much chance of making his voice outlast its echo, his body outrun its shadow. Sad, wasn't it?"[56]

Where much Greek effort continued, right down to the times of the last great Aristotelian commentators, Philoponus and Simplicius, in the +6th century, to be devoted to destructive criticism of rivals, there was a greater sense, in China, of the need to develop a common language. What this text in *Zhuangzi* helps to bring out is a related difference in the manner of dealing with disagreement. The reaction to Hui Shi's speculations in this chapter is not to confront, let alone refute him, but rather to pour scorn on his wasted effort. From the earliest Presocratic philosophers onwards, the preferred Greek style of reasoning, on the finite and infinite as on most other topics, was rather to attempt to disprove opponents by argument. But that, we can see, had the effect of prolonging dispute, rather than resolving it, and may thus have been one factor militating against the formation of an orthodoxy in Greece. The Chinese, by contrast, from Han times on especially, worked hard to secure one, and largely succeeded in elaborating an orthodoxy that served, as Sivin has shown, at once to legitimate imperial rule and to establish the role of the learned within the order of things.[57]

Of course the topic of the infinite, in its various manifestations is only one of many subjects that can be studied to help to throw light both on the social and

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[55] *Taiping yulan* 2, 6b–7a, discussed in Cullen, *Astronomy and Mathematics*, who speaks of an "age of polemic" when the *Huntian* system comes on the scene.

[56] *Zhuangzi* 33: 69–87.

[57] Sivin, "Yin Yang and the Five Phases."

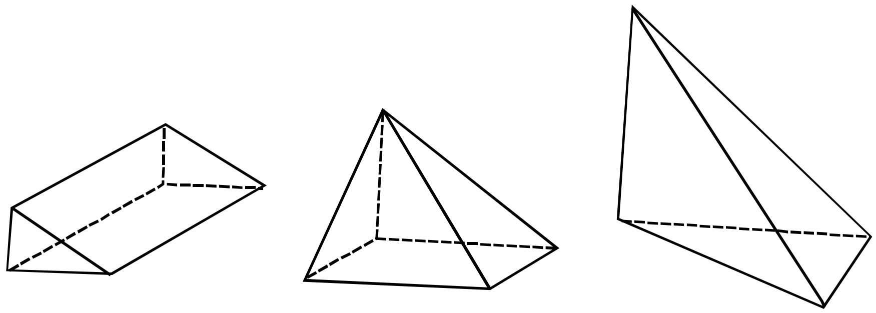


**Figure 1.**

professional institutions of science and philosophy in China and Greece, and on the effects of those institutions on the character of the science and philosophy produced. Moreover in the process there is much in the topic of the infinite, in its deployment in both China and Greece, that *remains* puzzling, and it is certainly as well to recognise that many of the specificities, of why this or that thinker proposed this or that use of the infinite in this or that context, are beyond the range of anything that could be called a hard-edged explanation. We shall no doubt never be able to explain Aristotle or Euclid or Hui Shi or Liu Hui in full, in the sense of explaining all the complex processes that contributed to their formulating the problems in the ways they did, and to their reacting to their predecessors in the precise manner in which that happened. But what we can hope to do is to use such specific topics as we have adumbrated here to explore the goals and the styles of argument of different Greek and Chinese thinkers at different junctures. The further investigation of those broader issues is, indeed, the subject of that larger collaborative project on which Professor Sivin and I are now engaged.

## Appendix

Liu Hui's problem is to find the formula for the volume of the figure called 陽馬 *yangma*, a pyramid with rectangular base and one lateral edge perpendicular to the base. He first shows that a *yangma* plus a 鱉臑 *bienao* (pyramid with right triangular base and one lateral edge perpendicular to the base) form 塹堵 *qiandu* (right prism with right triangular base) (Figure 1 sets out the shapes, Figures 2 and 3 show the result of adding the *bienao* BACE to the *yangma* BFECD: from Donald B. Wagner, "An Early Chinese Derivation of the Volume of a Pyramid: Liu Hui, Third Century AD," *Historia Mathematica* no. 6 [1979], pp. 164–88, at pp. 166ff).

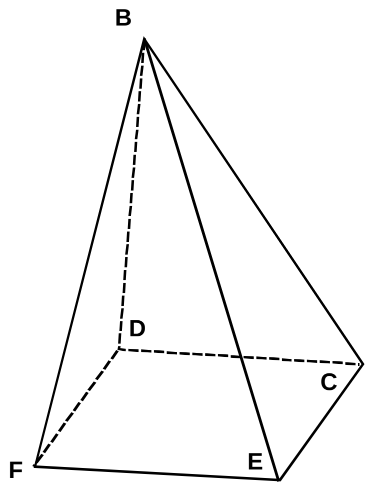


**Figure 2.**

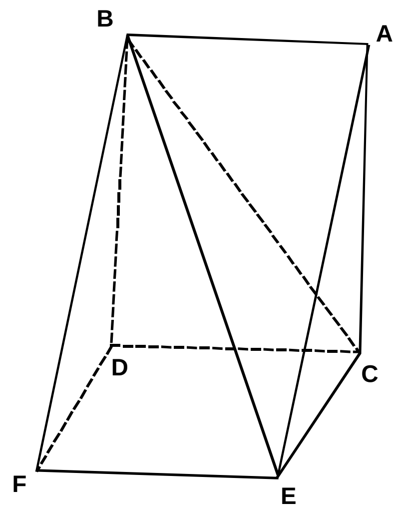


**Figure 3.**

But the volume of a *qiandu* is given as half length times breadth times height. So Liu Hui has to show that one *yangma* equals two *bienao* in order to give the formula for the volume of the *yangma* (namely one third length times breadth times height). This he does by first dividing the *yangma* and the *bienao* as in Figures 4 and 5. The *bienao* in Figure 4 (BACE) is divided into two *qiandu* (AGIJML and ILMJCP) and two *bienao* (BGIL and EPML).

The *yangma* in Figure 5 (BDFEC) is divided into one box (HILKNDRO), two *qiandu* (ILORCP and KLONFQ) and two *yangma* (BHILK and LOPEQ). But the sum of the two *qiandu* in the *bienao* (Fig. 4) can be seen to equal half that of the box and the two *qiandu* in the *yangma* (Fig. 5). So he must now show that the remaining *bienao* of the *bienao* are half the remaining *yangma* of the *yangma*. But they can be subdivided in exactly the same way as the original *bienao* and *yangma* were. Again part of the components thus distinguished can be seen to fulfil the requirement of the proportion, one *yangma* equals two *bienao*, leaving an even smaller remainder.

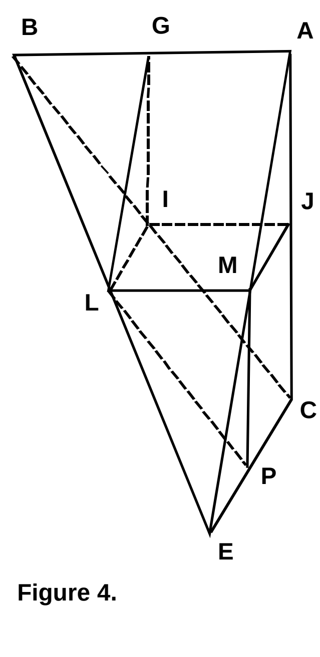


**Figure 4.**

If the process is continued, the series converges on the formula one *yangma* equals two *bienao*. While there are disagreements on points of detail between Wagner, “An Early Chinese Derivation,” Li Yan and Du Shiran, *Chinese Mathematics: A Concise History*, translated by John N. Crossley and Anthony W.-C. Lun (Oxford: Clarendon Press, 1987), pp. 70ff, and Karine Chemla, “Méthodes Infinitésimales en Chine et en Grèce Anciennes: les limites d’un parallèle,” in J. M. Salanskis and H. Sinaceur (eds.), *Le Labyrinthe du continu* (Berlin: Springer, 1992), pp. 31–46 at pp. 32ff, they adopt the same overall interpretation of this text.

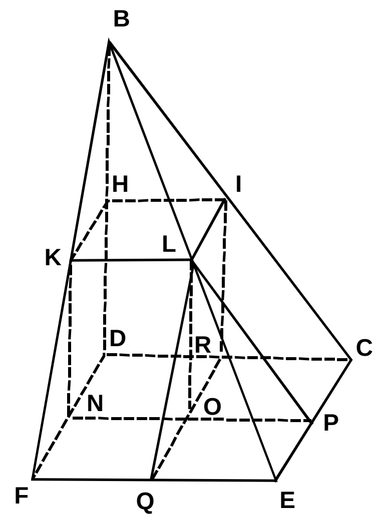


**Figure 5.**