

Time-dependent hebbian rules for the learning of templates for visual motion recognition

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Abstract. Experimental evidence suggests that the visual recognition of biological movements is based on learned spatio-temporal templates. Work in computational vision shows that movement recognition can be accomplished by recognizing temporal sequences of form or optic flow patterns. Recurrent neural networks with asymmetric lateral connections are one physiologically plausible way for the encoding of spatio-temporal templates. We demonstrate that time-dependent hebbian plasticity is suitable for establishing the required lateral connectivity patterns. We tested different hebbian plasticity rules and compared their efficiency and stability properties in simulations and by mathematical analysis. We found the most robust behavior for a learning rule that assumes a normalization of the total afferent synaptic connectivity that can be supported by each neuron. Consistent with psychophysical data our model learns the appropriate lateral connections after less than 30 stimulus repetitions. The resulting recurrent neural network shows strong sequence selectivity.

1 Introduction

The recognition of biological movements and actions is an important visual function. The perception of body as well as facial movements can be explained by the recognition of temporal sequences of form and optic flow patterns^[1,2]. A neural encoding of such temporal sequences can be realized in a physiologically plausible way by recurrent neural networks with asymmetric lateral connections^[2,3,4]. One possible mechanism for the learning of such connections is *time-dependent hebbian plasticity*. Recent experiments show that spikes initiated at the axon hillock can back-propagate into the dendrites, due to their active properties^[5]. These backpropagating signals provide information about previous activity states of the neuron. It also has been shown that the occurrence of LTP vs. LTD depends critically on the timing between pre- and postsynaptic spikes^[6,7]. Time-dependent synaptic plasticity is suitable for the realization of temporal sequence learning. In particular, it has been shown that spike-timing-dependent plasticity in cortical neurons can be related to the *temporal difference learning rule*^[8], where an activated synapse is strengthened or weakened dependent on the sign of the difference between two temporally subsequent output signals^[9,10].

Goal of this paper is to test several simple time-dependent Hebbian plasticity rules in the context of a model for the recognition of biological motion^[2]. This model consists of two parallel processing streams that are specialized for the analysis of form and optic flow information. Both pathways comprise hierarchies of neural feature detectors that extract features with increasing complexity along this hierarchy. The highest level of both processing streams consists of a recurrent neural network.

The units of this network encode body shapes, respectively temporary optic flow patterns, that arise during the course of the action. An example are the different body postures during a walking cycle (see Figure 1a). The motion stimulus activates the neurons of the network in sequence, inducing an activation pulse that propagates over the neural network (cf. Figure 2b).

After a short description of the network (*section 2*), we propose different Hebbian learning rules that can be used to establish the required form of lateral connectivity (*section 3*). After a mathematical analysis of the stability properties of these learning rules (*section 4*), we will present a number of simulations that compare the learning rules with respect to their efficiency and robustness (*section 5*)

2 Recurrent neural network model

The recurrent neural network that we used for the encoding of temporal sequences of body shapes or optic flow patterns has been originally proposed by Amari ^[11]. Sequence selectivity arises in this Hopfield-like network, if a suitable asymmetric form is chosen for the lateral connections. Signifying by $\mathbf{u}(t)$, the activity vector of all N_s neurons in the network, the dynamics is given by the differential equation:

$$\tau \left(\frac{d\mathbf{u}(t)}{dt} \right) + \mathbf{u}(t) = \tilde{\mathbf{W}}(t) \mathbf{f}(\mathbf{u}(t)) + \mathbf{s}(t) - h \quad (1)$$

The matrix $\tilde{\mathbf{W}}(t) = -w_l \mathbf{M} + \mathbf{W}^T(t)$ defines the synaptic strength of the lateral connections. It consists of a constant inhibitory part that ensures a sufficient level of lateral inhibition, and a second term that changes during learning. (\mathbf{M} is a matrix with only one elements, and w_l is a positive constant.) We used a step activation function with $f(z) = 1$ for $z > 0$ and $f(z) = 0$ otherwise. The time-dependent feed-forward inputs are given by the signal vector $\mathbf{s}(t)$. The time constant τ was chosen to be 200ms ¹ The positive parameter h defines the resting activity level.

For the simulations, we derived the input signal vector from the complete model for biological motion recognition ^[2] showing a walker as stimulus. Figure 2b shows that the input signal can be approximated by a localized positive activity pulse that moves along the neural network over time. In our implementation the recurrent network contained 20 neurons (Figure 1a). After the training of the lateral connections the network was tested with respect to its stability, sequence selectivity, and tuning with respect to stimulus speed and direction.

3 Learning Rules:

The most simple form of a time-dependent Hebbian rule can be written in matrix form by the differential equation:

$$\tau_w \frac{d\mathbf{W}}{dt} = \mathbf{s}(t) \mathbf{f}(\mathbf{u}^T(t - \Delta t)) \quad (2)$$

Consistent with the classical Hebbian postulate, the synaptic weight matrix \mathbf{W} changes with the product of the presynaptic activity vector $\mathbf{s}(t)$ and the postsynaptic activity vector $\mathbf{f}(\mathbf{u}(t))$, where the postsynaptic activity level enters the equation with the positive time delay Δt . It is crucial that the time constant τ_w of the learning rule is

¹ The differential equation was simulated using an Euler approximation with a step size of 65ms.

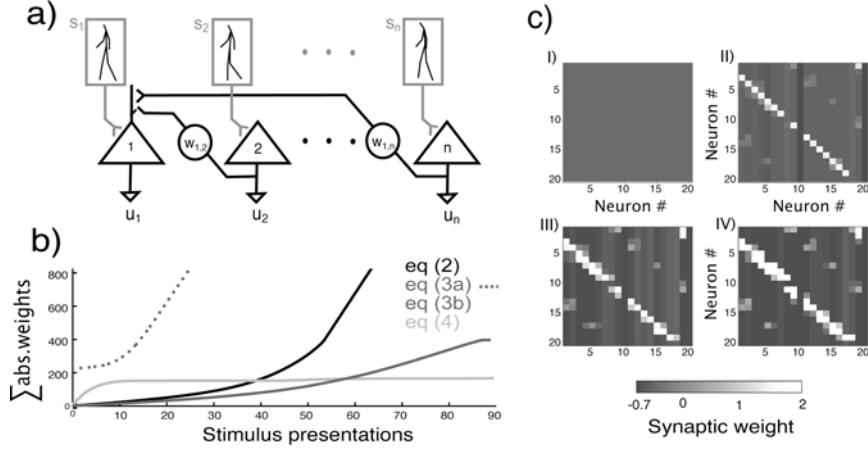


Fig. 1. (a) Illustration of the laterally connected network with units encoding body shapes arising during a walking cycle. (b) Sum of the absolute values of the elements of the weight matrix as a function of time for the learning rules (2), (3) (a, weights having positive and negative values, and b, weights being restricted to non-negative values), and for learning rule (4). (c) Plot of the weight matrix learned with rule (4). (I) Initial weight matrix with all elements set to zero. (II), (III) and (IV) show the weight matrix after 15, 30 and 45 stimulus repetitions.

much larger than the time constant τ of the network dynamics and the duration of the encoded actions.

It is well-known that the simple Hebbian rule is unstable. This implies that the elements of the weight matrix tend to become unbounded after several stimulus presentations^[12]. This can be prevented by introduction of competition between the different synapses by weight normalization. One possibility to realize such competition is to modify learning rule (2) by subtraction of a term that depends on the sum of the input signal vector $\mathbf{s}(t)$. Let \mathbf{m} be an N_s -dimensional vector whose components are all one, then a modified learning rule is given by:

$$\tau_w \frac{d\mathbf{W}}{dt} = \left[\mathbf{s}(t) - \frac{\mathbf{m}^T \mathbf{s}(t) \mathbf{m}}{N_s} \right] \mathbf{f}(\mathbf{u}^T(t - \Delta t)) \quad (3)$$

It is shown in section 4 that this form of normalization imposes a rigid constraint on the sum of the weights in each column of the matrix \mathbf{W} . This implies that increases of some weights lead to decreases of others, stabilizing the behavior of the weights during learning. We tested another version of this learning rule where we restricted the weights to non-negative values.

Another way to stabilize the learning process is to impose a constraint on the sum of synaptic weights that is supported by each neuron. A limitation of the maximum number of synapses that is supported by individual neurons seems to be suggested by electrophysiological data^[13]. A learning rule that implements this constraint can be written in matrix form:

$$\tau_w \frac{d\mathbf{W}}{dt} = \left[\mathbf{s}(t) \mathbf{m}^T - \mathbf{m} \mathbf{m}^T \mathbf{W}(t) + c \mathbf{M} \right] \mathbf{D}_u \quad (4)$$

where \mathbf{D}_u is a diagonal matrix with the elements $D_{u,kk} = f(u_k(t-\Delta t))$. The positive constant α determines the sum of the weights in each column of the weight matrix \mathbf{W} (see section 4). In the final implementation the weights were additionally constrained to be non-negative.

4 Stability analysis

For the chosen simple learning rules the effect of the normalization term can be analyzed mathematically if the positivity constraint for the weights is dropped. We assume in the following an idealized input signal vector $\mathbf{s}(t)$ that is given by a positively activated region with constant shape that moves along the neural network².

4.1 Stability analysis for learning rule (3):

By multiplying both sides of equation (3) with \mathbf{m} we can derive a differential equation for the vector $\mathbf{v}^T(t) = \mathbf{m}^T \mathbf{W}(t)$ that defines the sums of the weights in the columns of the weight matrix. The sum $\sigma = \mathbf{m}^T \mathbf{s}(t)$ does not depend on time because of the constant shape of the moving input peak. The resulting differential equation

$$\tau_w \frac{d\mathbf{m}^T \mathbf{W}}{dt} = \tau_w \dot{\mathbf{v}}^T(t) = \left[\sigma - \frac{N_s}{N_s} \sigma \right] \mathbf{f}(\mathbf{u}^T(t-\Delta t)) = \mathbf{0}^T \quad (5)$$

implies $\mathbf{v}^T = \mathbf{m}^T \mathbf{W} = \text{const}$. This implies that the column vectors of the weight matrix remain within hyperplanes with normal vector \mathbf{m} . Still the weights can diverge tangentially to these hyperplanes. If the weights are additionally constrained to positive values they remain bounded within the interval $0 \leq W_{mn} \leq \max(\mathbf{m}^T \mathbf{W}(0))$.

4.2 Stability analysis of learning rule (4):

Multiplication of equation (4) with \mathbf{m} yields:

$$\tau_w \frac{d\mathbf{m}^T \mathbf{W}}{dt} = \tau_w \dot{\mathbf{v}}^T(t) = \left[(\sigma + \alpha N_s) \mathbf{m}^T - N_s \mathbf{v}^T(t) \right] \mathbf{D}_u \quad (6)$$

Let T signify the cycle time of the walking stimulus. The temporal dynamics of the learning process is much slower than the activation dynamics of the network ($\tau_w \gg T, \tau$), so that we can average the dynamical equation (6) over time^[14]. If the activation of the network is close to a form-stable solution that moves with the stimulus signal along the network we obtain with the time-averaged signals

$$\mathbf{y}(t) = \frac{1}{T} \int_t^{t+T} \mathbf{v}(t) dt \quad \frac{1}{T} \int_t^{t+T} \mathbf{f}(\mathbf{u}(t-\Delta t)) dt \approx f_0 \mathbf{m} \quad (7)$$

where the constant f_0 fulfills $0 < f_0 < 1$. We obtain the following approximative dynamical equation for the averaged quantities:

$$\tau_w \frac{d\mathbf{y}}{dt} = \tau_w \dot{\mathbf{y}}(t) = \left[(\sigma + \alpha N_s) \mathbf{m} - N_s \mathbf{y}(t) \right] f_0 \quad (8)$$

² Because walking is a periodic pattern the input signal is also periodic, and the activation distribution is spatially periodic.

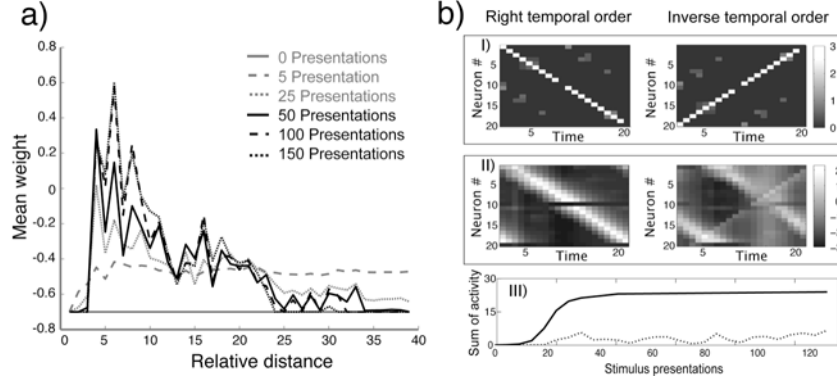


Fig. 2. (a) Development of the interaction kernel over time for learning rule (4). The weight kernel develops quickly with only marginal changes after 25 stimulus repetitions. (b) Development of direction selectivity for learning rule (4): (I) shows the stimulus signal $s(t)$ for the stimulus in normal and inverse temporal order. (II) illustrates the output activity after 25 stimulus presentations, and (III) the sum of the activity of all neurons for the correct (solid line) and the inverse temporal order (dashed line).

The last dynamics is stable because N_s and f_0 are always positive. The stationary solution can be easily computed to be $\mathbf{y}^* = \mathbf{m} (\sigma + \alpha N_s) / N_s$. The stable sum of weights of the columns of \mathbf{W} depends on the constant α , and on the strength of the input signal $\mathbf{s}(t)$. Again the weight vector can change orthogonal to the vector \mathbf{m} without affecting the previous dynamical equations. If the weights are restricted to positive values the learning rules produce stable results.

5 Simulation results

The sum of the absolute values of the weight matrix learned with different learning rules is shown in Figure 1b. As predicted, the simple Hebbian learning rule (2) leads to unbounded growth of the weight vector. No direction selectivity arises in the network because all units become activated after some time. Also learning rule (3) is suboptimal. If the weights were not constrained to positive values, consistent with the mathematical analysis, the sum of the weights remains constant, but the absolute values of the weights grow and become unbounded, since positive and negative weights compensate each other in the sum (curve 3a). Only if the weights were restricted to non-negative values, the learning rule converges to a stable weight kernel, after a large number of gait cycles (curve 3b). For learning rule (4) we found a much faster stabilization of the learned weight distribution. After only eight gait cycles the sum of the weights reaches its plateau.

Figure 2a presents the learned weight distributions using rule (4) with additional positivity constraint for the W_{ij} . The learned weight distribution is periodic with respect to the neuron number, reflecting the periodic nature of the stimulus. The figure shows the learned lateral weight kernel as function of the index difference between the connected neurons. This kernel can be obtained by averaging the weights that connect neurons with the same relative distance in the network over all neurons.

After 25 stimulus presentations the form of the kernel remains quite stable. This fast learning is compatible with psychophysical data demonstrating substantial

improvements during the learning of novel biological motion patterns after less than 30 stimulus presentations (Jastorff, Kourtzi & Giese, submitted).

Figure 2b illustrates that during the learning process substantial sequence selectivity arises. After 25 presentations we find strong dependence on temporal order. If the stimulus (panel I) is presented in reverse order much weaker activity arises in the network than for the correct temporal order of the stimulus frames (panel II). The proposed learning rule is thus suitable for establishing sequence selectivity with a small number of training trials. Panel (III) shows that the activity level for correct temporal order substantially increases already after 10 stimulus presentations and reaches a plateau after about 40 presentations. The activity for the reverse temporal order stimulus remains at low levels throughout the whole training.

6 Conclusion

We have discussed different time-dependent Hebbian learning rules to establish sequence selectivity in a recurrent neural network for biological motion recognition. Most efficient were learning rules that enforce a constant sum of synaptic weights that are supported by each neuron, combined with a lower bound for the connection strength. Both assumptions seem physiologically plausible, because usually excitatory synapses cannot change into inhibitory ones, and because neurons can only supply transmitter for a limited number of synapses^[13]. Given the high number of recurrent connections in the visual cortex, it seems plausible that lateral connections play an important role, potentially also for the realization of sequence selectivity.

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